PRODUCTS AND PRODUCTIVITY

by

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Abstract

When firms make decisions about which product to manufacture at a more disaggregated level than observed in the data, measured firm productivity will reflect both true differences in productivity and non-random decisions about which products to manufacture. This paper examines a model of industry equilibrium where firms endogenously sort across products. We use the model to characterize the direction and magnitude of the resulting bias in productivity and to trace the implications for evaluating the aggregate effects of policy reforms such as industry deregulation. The endogenous sorting of firms across products provides a new source of reallocation and leads to biased measures of deregulation's impact on firm and aggregate productivity.

Keywords: Product choice, Productivity measurement, Firm heterogeneity, Industry deregulation

JEL classification: L11, D21, L60

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1. Introduction

Measurement of firm productivity is one of the core empirical challenges in both micro and macroeconomics and one that lies at the heart of an array of policy debates, ranging from the impact of information technology to the consequences of industry deregulation. This paper argues that the endogenous sorting of firms across products is an important and hitherto largely neglected source of bias in productivity measurement. Since firms typically choose products at a more disaggregated level than is observed in plant and firm-level datasets, measured firm productivity reflects both true differences in firm productivity and firms' non-random decisions about which products to make. The paper develops a model of industry equilibrium to characterize the direction and magnitude of the bias in measured productivity and to trace the implications for macroeconomic variables such as aggregate productivity and the firm-size distribution. In the model, changes in demand or barriers to entry generate changes in aggregate productivity as a result of both firm entry and exit and the endogenous re-sorting of firms across products.

The paper is related to the large and growing literature that uses plant or firm-level data to analyze the microeconomics and macroeconomics of productivity. This literature stretches across fields as diverse as macroeconomics (Bloom 2006 and Rossi-Hansberg and Wright 2006), international trade (Pavcnik 2002, Trefler 2004, Tybout 2003), development economics (van Biesebroeck 2005, Banerjee and Munshi 2004) and industrial economics (Dunne *et al.* 1989, Levinsohn and Petrin 2003, Aghion *et al.* 2005 and Griffith *et al.* 2006). The vast majority of the firm and plant-level datasets used in this literature (e.g. the United Kingdom manufacturing census) report the "main industry" of a firm or plant, but do not report all of the industries or products within industries in which the firm or plant is active.[1] This main industry is typically one of a just a few hundred industries into which manufacturing has been split.

While the use of these plant and firm-level datasets enables a number of concerns about changes in composition within industries to be addressed, the classification of plants and firms according to "main industry" raises the problem of their endogenous sorting across products within industries. Here, we characterize this deficiency by examining data from the U.S. Longitudinal Research Database (LRD). In contrast to the manufacturing censuses of many other countries, the U.S. data report all of the five-digit SIC (Standard Industrial Classification) products a firm manufactures as well as its main four-digit SIC industry. As a result, we can isolate single-product plants within each industry. Analysis of these plants reveals considerable heterogeneity across five-digit producers within four-digit industries in terms of both labor and

[1] We use the term "industry" to refer to the level at which output and factor inputs are typically observed in the data and the term "product" to refer to the more disaggregated level at which firm decisions are actually made. Census of Production datasets usually allocate a firm or plant to a "main industry" based on the product or set of products that accounts for the majority of its sales.

total factor productivity (TFP).

To further explore the bias in productivity measures that results from the endogenous sorting of firms across products, and to examine the underlying determinants of product choice, the paper develops a theoretical model of industry equilibrium. Existing theories of industry dynamics emphasize either learning about a true unknown value of productivity (as in Jovanovic 1982) or stochastic realizations of true productivity (as in Hopenhayn 1992, Ericson and Pakes 1995, Melitz 2003 and Bernard, Redding and Schott 2006). The framework in this paper is based on Melitz's (2003) model of industry equilibrium. Melitz's framework substantially simplifies industry dynamics by assuming a monopolistically competitive industry structure where firms produce varieties of a single product, by assuming that firm productivity is a parameter which is drawn from a fixed distribution at the point of entry, and by assuming that firms face a constant, exogenous probability of death thereafter. Into this structure we introduce firm choice between two heterogeneous products, which have different production techniques and enter demand asymmetrically, so that their relative price is determined endogenously in general equilibrium. We believe this to be the simplest framework for understanding the impact of product choice on productivity measurement. It captures the endogenous sorting of firms across products, while remaining tractable enough to quantify the direction and magnitude of the bias in productivity measurement. It also allows analysis of the implications of endogenous product choice for the comparative static properties of the model.

The remainder of the paper is structured as follows. Section 2 discusses the bias in productivity measurement that results from the endogenous sorting of firms across products, relates our analysis to other measurement problems stressed in the productivity literature, and presents some empirical evidence on the quantitative importance of the bias due to the self-selection of firms into heterogeneous products. Section 3 develops the theoretical model, while Section 4 solves for industry equilibrium. Section 5 examines the properties of general equilibrium and derives the direction and magnitude of the bias in standard productivity measures. Section 6 examines the implications of an exogenous policy reform, in the shape of a reduction in barriers to entry, for product choice, measured firm productivity and aggregate industry productivity. Section 7 concludes. An appendix at the end of the paper collects together proofs and technical derivations.

2. Framing the Problem

The existing literature on productivity measurement has focused largely on three broad classes of problems. First, there is the "exit selection problem" of the nonrandom survival of firms and plants. A large amount of empirical research has demonstrated that exiting plants are systematically less productive than survivors (e.g., Dunne *et al.* 1989, Baily *et al.* 1992, Olley and Pakes 1996 and Foster et al.

2001). Second, there is the "endogeneity problem" that the factor input choices of surviving firms are partly determined in response to firm productivity. Therefore, if a production function is to be estimated, the simultaneity of factor input choices must be controlled for (Marschak and Andrews 1944, Olley and Pakes 1996, Blundell and Bond 1998 and 2000 and Levinsohn and Petrin 2003). Third, there is a "specification problem" of the correct functional form of the production technology and the market structure assumptions needed to identify productivity separately from the influence of market power. By itself, this third issue encompasses a wide range of research on a variety of issues including Caves *et al.* (1982a,b), Hall (1988), Roeger (1995), Klette and Griliches (1996) and Levinsohn and Melitz (2006) among many others.

The role of these three classes of problems in the measurement of productivity can be illustrated by considering the following expression for the expected productivity φ of a firm conditional on a vector of observed characteristics X:

$$E\left(\varphi|X\right) = \underbrace{\left(1 - G\left(\varphi^{*}|X\right)\right)}_{\text{Term } A} \underbrace{E\left(\varphi|X, \varphi > \varphi^{*}\right)}_{\text{Term } B}$$

where Term A captures the probability that productivity exceeds a threshold φ^{*} below which firms exit, and Term B captures expected productivity conditional on firm survival. The selection problem relates to correctly controlling for Term A, while the endogeneity and specification problems relate to adequately modelling Term B.

Our analysis emphasizes an additional and neglected challenge in measuring productivity. Since firms typically choose products at a more disaggregated level than is observed in the data, there is a "product selection problem" due to the endogenous sorting of firms across products. Firms choose which of a number of heterogeneous products to manufacture based on their unobserved characteristics, and so measured productivity differences across firms reflect both variation in firms' true productivity and their non-random decisions about what products to make. This additional challenge can also be illustrated using the framework above. Taking the simplest case of two products within an industry, the expected productivity of a firm conditional on a vector of observed characteristics can be written as:

$$E\left(\varphi|X\right) = \underbrace{\left(1 - G\left(\varphi^{*}|X\right)\right)}_{\text{Term } A} \left[\begin{array}{c} \underbrace{\left(\frac{G\left(\varphi^{**}|X\right) - G\left(\varphi^{*}|X\right)}{1 - G\left(\varphi^{*}|X\right)}\right)}_{\text{Term } C} \underbrace{E\left(\varphi|X, \varphi^{*} < \varphi < \varphi^{**}\right)}_{\text{Term } D} \\ + \underbrace{\left(\frac{1 - G\left(\varphi^{**}|X\right)}{1 - G\left(\varphi^{*}|X\right)}\right)}_{\text{Term } E} \underbrace{E\left(\varphi|X, \varphi > \varphi^{**}\right)}_{\text{Term } F} \end{array} \right]$$

where Term A again captures the probability that productivity exceeds the threshold φ^{*} below which firms exit; Terms C and E capture the probabilities that a firm makes each product and they depend on firm productivity; Terms D and F capture expected productivity conditional on making each product. In the example considered here and fleshed out below, we assume that firms with productivities between the exit threshold φ^* and another product selection threshold φ^{**} make one product, while firms with productivities above the product selection threshold make the other product.

The product selection problem relates to the fact that product choice depends on productivity, and so Terms C and E are systematically correlated with Terms D and F. As a result, there is an aggregation problem because the industry includes heterogeneous products, but there also exists a product-selection problem because firms with specific characteristics are self-selecting into particular products. The exit selection problem emphasized in existing research remains present through Term A. Similarly, the endogeneity and specification problems continue to apply through Terms D and F. But there is now an additional bias introduced into productivity measurement as a result of the endogenous sorting of firms across products that is captured by Terms C and E.

The empirical relevance of these problems can be illustrated using information from the U.S. Longitudinal Research Database (LRD). As in many other manufacturing censuses, the LRD reports the main four-digit Standard Industry Classification (SIC) industry of a plant. Unlike most other censuses, however, the LRD also reports each of the five-digit SIC products produced by the plant as well as their share of total plant shipments. Because inputs are only observed for the plant as a whole, productivity by plant-product cannot be computed. We can, however, make use of the fact that the Census contains a large number of single-product plants. These plants can be grouped together for the purposes of computing productivity for each producer of a five-digit SIC product. We can then examine variation in productivity across plants within the same four-digit SIC industry.

We assess the importance of product-level variations in plant productivity by regressing the TFP and labor productivity of single-product manufacturing plants on a series of progressively more disaggregated SIC fixed effects.[2] We report adjusted-R^2 to control for the different number of fixed effects being estimated at the two-, three-, four- and five-digit SIC levels. The first row of column 1 reports the adjusted-R^2 of an OLS regression of plant TFP on two-digit SIC fixed effects. The second, third and fourth columns of the table report the adjusted-R^2 from OLS regressions with three-, four- and five-digit SIC fixed effects, respectively. In the last specification, the five-digit SIC fixed effects capture within-industry variation in productivity across plants specializing in different products. As in the model developed below, this variation reflects both differences in production technology across products as well as firms' non-random decisions about which product to manufacture based on their heterogeneous characteristics.

As indicated in the table, the inclusion of the five-digit product fixed effects in-

[2] We consider a simple TFP measure assuming a Cobb-Douglas production technology: the difference between log output and log inputs using the mean cost shares for each input across plants in the same product market.

	Industry Fixed Effects			
Productivity Measure	2-Digit	3-Digit	4-Digit	5-Digit
TFP	0.11	0.17	0.20	0.24
Labor Productivity	0.23	0.33	0.38	0.45

Notes: Table reports adjusted R^2s of OLS regressions of noted produtivity measure on SIC fixed effects at four levels of aggregation: two-digit, three-digit, four-digit and five-digit SIC categories. Sample is restricted to 93,246 single five-digit SIC product U.S. manufacturing plants in 1997. TFP is measured as the difference between log output and log inputs using the mean cost shares for each input across firms in the same product market. Inputs are non-production workers, production workers, capital and materials. Labor produtivity is plant output divided by plant employment.

Table 1: Explaining Variation in U.S. Manufacturing Plant Productiivity

creases the regression adjusted-R^2 by around 20 percent and 18 percent vis a vis the specification with four-digit SIC fixed effects for TFP and labor productivity, respectively. By comparison, the inclusion of the four-digit industry fixed effects increases the regression adjusted-R^2 by around 18 percent and 15 percent vis a vis three-digit SIC fixed effects for TFP and labor productivity, respectively. Therefore, the variation in either measure of productivity across five-digit products within four-digit industries is as great, if not greater, than the variation in productivity across four-digit industries within three-digit sectors. Taken together, these results provide clear evidence of heterogeneity in productivity across plants specializing in different five-digit SIC products within four-digit industries. As is clear from the table, there is also substantial heterogeneity in productivity within five-digit products, and both sources of heterogeneity are captured in the theoretical model developed below.

3. Theoretical Model

In this section, we develop a theoretical model of industry equilibrium in which heterogeneous firms endogenously sort across products, thus introducing a bias into measures of firm productivity. We consider a single industry within which consumers and firms choose whether to consume and produce varieties of two distinct products.[3] To keep the analysis as tractable as possible, we assume that consumer preferences between the two products can be well represented with the following CES utility function:

$$U = \left[aC_1^{\nu} + (1-a)C_2^{\nu}\right]^{1/\nu}. \tag{1}$$

where a captures the relative strength of preferences for each product, and we assume that the products are imperfect substitutes with elasticity of substitution $\psi = \frac{1}{1-\nu} >$

[3]It is straightforward to embed this framework in a multi-industry model or to allow a finite number of distinct products within the industry. The model developed here is the simplest framework within which to demonstrate the importance of firms' choice between heterogeneous products in influencing measured firm and industry outcomes.

1. Firms produce horizontally differentiated varieties of their chosen product. C_i is therefore a consumption index defined over varieties ω of each product i:

$$C_i = \left[\int_{\omega\in\Omega_i} q_i(\omega)^{\rho} d\omega\right]^{1/\rho}, \qquad P_i = \left[\int_{\omega\in\Omega_i} p_i(\omega)^{1-\sigma} d\omega\right]^{1/1-\sigma}. \tag{2}$$

where $\{\Omega_i\}$ is the set of available varieties in market i, P_i is the price index dual to C_i, and $\sigma = \frac{1}{1-\rho} > 1$ is the elasticity of substitution between varieties of the same product. We make the natural assumption that varieties of the same product are more easily substitutable than different products, so that $\sigma > \psi$.

Consumer expenditure minimization yields the following expression for equilibrium expenditure (equals revenue, $r_i(\omega)$) on a variety:

$$r_i(\omega) = R_i\left(\frac{p_i(\omega)}{P_i}\right)^{1-\sigma} = \alpha_i\left(\mathcal{P}\right)R\left(\frac{p_i(\omega)}{P_i}\right)^{1-\sigma} \tag{3}$$

which is increasing in aggregate expenditure (equals aggregate revenue $R = R_1 + R_2 = \int_{\omega\in\Omega_1} r_1(\omega)d\omega + \int_{\omega\in\Omega_2} r_2(\omega)d\omega$), increasing in the share of expenditure allocated to product i, $\alpha_i(P_2/P_1) = \alpha_i(\mathcal{P})$, decreasing in own variety price, $p_i(\omega)$, and increasing in the price of competing varieties as summarized in the price index, P_i.

With CES utility, the share of expenditure allocated to product 1 is increasing in the relative price of product 2, $\mathcal{P} = P_2/P_1$ (since $\psi > 1$), and increasing in the relative weight given to product 1 in consumer utility, a:

$$\alpha_1\left(\mathcal{P}\right) = \left[1 + \left(\frac{1-a}{a}\right)^{\psi}\mathcal{P}^{1-\psi}\right]^{-1}, \qquad \alpha_2\left(\mathcal{P}\right) = 1 - \alpha_1\left(\mathcal{P}\right). \tag{4}$$

3.1. Production

As well as entering demand in different ways, the products have different production technologies. Labor is the sole factor of production and is supplied inelastically at its aggregate level L, which also indexes the size of the economy. The production technology follows Melitz (2003) in that variable cost is assumed to depend on heterogeneous firm productivity. We differ in that we allow for different products and hence endogenous product choice within the industry. The labor required to produce q_i units of a variety in product market i is given by:

$$l_i = f_i + \frac{b_i q_i}{\varphi} \tag{5}$$

so that the variable cost of production depends on b_i, which is common to all firms, as well as on the firm-specific productivity, φ.[4]

[4]The assumption that fixed costs of production are independent of productivity captures the idea that many fixed costs, such as building and equipping a factory with machinery, are unlikely to vary substantially with firm productivity. As long as fixed costs are less sensitive to productivity than variable costs, there will be endogenous selection on productivity in firms' exit and product choice decisions.

Products differ in terms of both their fixed and variable costs of production. We assume that product 2 has a lower variable cost of production than product 1: without loss of generality, we normalize b_1 to unity and denote the relative variable costs of product 2 by $b < 1$, so that $b_2 = b < b_1 = 1$. To manufacture the lower variable cost product 2, we assume that a firm must incur a higher fixed cost: $f_2 > f_1$. This assumption is natural if a lower variable cost reflects a higher level of technology and a firm must incur a greater fixed cost in order to manufacture a higher technology product. Nonetheless, we also consider the alternative possibility where the product with the lower variable cost has the lower fixed cost. Even in this case, both products are produced in equilibrium: the products are imperfect substitutes in utility (1) and the marginal utility derived from a product approaches infinity as consumption tends to zero. Therefore, the relative price indices for the two products $\mathcal{P}$ adjust to ensure that in equilibrium both products are produced.

Fixed production costs and consumer love of variety imply that each firm manufacturing a product chooses to produce a unique variety of that product. Profit maximization yields the standard result that equilibrium prices are a constant mark-up over marginal cost, with the size of the mark-up depending on the elasticity of substitution between varieties:

$$p_i(\varphi) = \left(\frac{\sigma}{\sigma - 1}\right) \frac{wb_i}{\varphi}. \tag{6}$$

We choose the wage as the numeraire so that $w = 1$. Using this choice of numeraire and the pricing rule in the expression for revenue above, equilibrium firm revenue and profits are:

$$\begin{aligned} r_i(\varphi) &= \alpha_i(\mathcal{P}) R \left(P_i \rho \frac{\varphi}{b_i}\right)^{\sigma-1} \\ \pi_i(\varphi) &= \frac{r_i(\varphi)}{\sigma} - f_i. \end{aligned} \tag{7}$$

One property of equilibrium revenue that will prove useful below is that the relative revenue of two firms with different productivity levels in the same product market depends solely on their relative productivity: $r_i(\varphi'') = (\varphi''/\varphi')^{\sigma-1} r_i(\varphi')$. Similarly, the relative revenue of two firms with different productivity levels in different product markets depends on their relative productivities, the relative variable cost of making the two products, the relative expenditure share devoted to the two products, and relative price indices:

$$r_2(\varphi'') = \left(\frac{1 - \alpha_1(\mathcal{P})}{\alpha_1(\mathcal{P})}\right) \left[\left(\frac{\varphi''}{\varphi'}\right) \mathcal{P} \frac{1}{b}\right]^{\sigma-1} r_1(\varphi'). \tag{8}$$

3.2. *Industry Entry and Exit*

To enter the industry (and produce either product), a firm must pay a fixed entry cost, $f_e > 0$, which is thereafter sunk. After paying the sunk cost, the firm draws its productivity, φ, from a distribution, $g(\varphi)$, with corresponding cumulative distribution $G(\varphi)$. This formulation captures the idea that there are sunk costs of entering an industry and that, once these costs are incurred, some uncertainty regarding the nature of production and firm profitability is realized. Firm productivity is assumed to remain fixed thereafter, and firms face a constant exogenous probability of death, δ, which we interpret as due to *force majeure* events beyond managers' control.[5]

A particularly tractable productivity distribution, which provides a good approximation to observed firm-level productivity data, is the Pareto distribution: $g(\varphi) = zk^z\varphi^{-(z+1)}$. The parameter $k > 0$ corresponds to the minimum value of productivity in the industry, while $z > 0$ determines the skewness of the distribution, and a finite variance of log productivity with a Pareto distribution requires $z > \sigma - 1$. Although we develop our results analytically without assuming a particular form for the productivity distribution, we consider a Pareto productivity distribution when we calibrate the model and analyze industry deregulation.[6]

After entry, firms decide whether to begin producing in the industry or exit. To concentrate on the endogenous sorting of firms across products, and to abstract from issues of multi-product firms, we begin by assuming that managerial diseconomies of scope are such that a firm can only produce one product. In the appendix, we relax this assumption in order to introduce multi-product firms, and discuss the additional biases in productivity measurement that result. But in our baseline analysis, if a firm decides to become active in the industry, it must choose which of the two products to make. Therefore, the value of a firm with productivity φ is the maximum of 0 (if the firm exits) or the stream of future profits from producing one of the two products discounted by the probability of firm death:

$$v(\varphi) = \max\left\{0, \frac{1}{\delta}\pi_1(\varphi), \frac{1}{\delta}\pi_2(\varphi)\right\}. \tag{9}$$

[5]Firm death ensures steady-state entry into the industry. New entrants make an endogenous exit decision, since their decision whether or not to produce in the industry depends on their productivity draw φ from the distribution $g(\varphi)$. Together with fixed production costs, this will generate the result that exiting firms are on average less productive than surviving firms. For incumbent firms, the probability of death δ is independent of productivity. This assumption can be relaxed by allowing firm productivity to evolve stochastically after entry (e.g. Hopenhayn 1992). While this would achieve greater realism, it would not change the qualitative results below on the importance of endogenous product choice for measured firm and industry productivity, and would come at the cost of a substantial increase in the complexity of the industry dynamics.

[6]See Axtell (2001) for empirical evidence that the Pareto distribution approximates the observed distribution of firm sizes.

3.3. Product Choice

Firms decide which product to make based on their realized productivity, taking as given aggregate variables such as the price indices. From our expression for equilibrium profits above, firms with zero productivity have negative post-entry profits and profits are monotonically increasing in productivity. Fixed production costs mean that there is a positive value for productivity below which negative profits would be made. Firms drawing a productivity below this **zero-profit productivity cutoff**, φ^*, exit the industry immediately.

Since product 2 has a higher fixed product cost than product 1, firms with zero productivity would make the largest losses from producing product 2:

$$0 > \pi_1(0) = -f_1 > \pi_2(0) = -f_2. \tag{10}$$

Since profits for each product are monotonically increasing in productivity, a necessary condition for both products to be produced is that profits from product 2 increase more rapidly with productivity than those from product 1:

$$\frac{d\pi_2/d\varphi}{d\pi_1/d\varphi} = \Gamma \equiv \left(\frac{1-a}{a}\right)^{\psi} \left(\frac{1}{b}\right)^{\sigma-1} \mathcal{P}^{\sigma-\psi} > 1 \tag{11}$$

where the *relative* rate at which profits increase with productivity is independent of productivity, and depends instead on parameters, such as the demand-shifter a and the variable cost parameter b, as well as endogenous relative price indices, $\mathcal{P}$.

The *sufficient* condition for both products to be produced is that profits are positive in each product market and exceed those in the other product market over a range of productivities:

$$\begin{aligned} \pi_1(\varphi) &> 0 \quad \text{and} \quad \pi_1(\varphi) > \pi_2(\varphi) \quad \text{for } \varphi \in \Phi_1 \subset (0, \infty) \\ \pi_2(\varphi) &> 0 \quad \text{and} \quad \pi_2(\varphi) > \pi_1(\varphi) \quad \text{for } \varphi \in \Phi_2 \subset (0, \infty) \end{aligned} \tag{12}$$

which requires the profit functions for the two products to intersect at a value for productivity where positive profits are made, as shown graphically in Figure 1. As we show formally when we solve for general equilibrium, consumers' taste for both products implies that relative prices, $\mathcal{P}$, will adjust to ensure that the conditions in equation (12) are satisfied even if product 2 has both a higher fixed and variable cost. The point at which the two profit functions intersect defines the **product-indifference productivity cutoff**, φ^{**}, at which a firm is exactly indifferent between the two products.

The higher fixed cost for product 2 and the requirement that the two profit functions intersect at a value for productivity where positive profits are made together imply that product 1 will be produced by the lowest productivity firms that are active in the industry and product 2 will be produced by higher productivity firms.

The zero-profit productivity cutoff determining the lowest level of productivity where product 1 is produced is given by:

$$r_1(\varphi^*) = \sigma f_1, \tag{13}$$

while the product-indifference productivity cutoff defining the lowest level of productivity where product 2 is produced is defined by:

$$\frac{r_2(\varphi^{**})}{\sigma} - f_2 = \frac{r_1(\varphi^{**})}{\sigma} - f_1. \tag{14}$$

Firms drawing a productivity below φ^{**} but above φ^* will make product 1, while those drawing a productivity above φ^{**} will make product 2.

Note that the key difference between our framework and existing models of industry equilibrium is that we allow firms to choose between heterogeneous products within the industry. In particular, the special case of our framework where $a = 1 - a$, $\psi = \sigma$, $f_1 = f_2$ and $b_1 = b_2$ corresponds to the Melitz (2003) model (under these parameter restrictions, the two frameworks are equivalent up to a re-scaling of utility by a constant equal to $a^{1/\nu}$). With $a = 1 - a$ and $\psi = \sigma$, the two products receive equal weight in consumers' utility, and the elasticity of substitution across products is the same as the elasticity of substitution across varieties within products. With $f_1 = f_2$ and $b_1 = b_2$, there are no differences in production technology across products. Therefore, taking these two sets of properties together, the model collapses to the special case of many varieties of a single product within the industry. In contrast to this special case, our framework allows for heterogeneity in both demand and production technology across products, and we discuss below the respective contributions of these sources of heterogeneity to biases in productivity measurement.

3.4. Free Entry

From the characterization of entry and product choice in the previous sections, the *ex ante* probability of successful entry into the industry is $[1 - G(\varphi^*)]$, with the *ex ante* probability of producing product 1 given by $[G(\varphi^{**}) - G(\varphi^*)]$, and the *ex ante* probability of producing product 2 given by $[1 - G(\varphi^{**})]$. The *ex post* productivity distribution for each product, $\mu_i(\varphi)$, is conditional on successful entry and product choice and is a truncation of the *ex ante* productivity distribution, $g(\varphi)$:

$$\begin{aligned} \mu_1(\varphi) &= \begin{cases} \frac{g(\varphi)}{G(\varphi^{**}) - G(\varphi^*)} & \text{if } \varphi \in [\varphi^*, \varphi^{**}) \\ 0 & \text{otherwise} \end{cases}, \\ \mu_2(\varphi) &= \begin{cases} \frac{g(\varphi)}{1 - G(\varphi^{**})} & \text{if } \varphi \in [\varphi^{**}, \infty) \\ 0 & \text{otherwise} \end{cases}. \end{aligned} \tag{15}$$

In equilibrium we require the expected value of entry in the industry, v_e, to equal the sunk entry cost, f_e. The expected value of entry is the *ex ante* probability of making product 1 times expected profitability in product 1 until death plus the *ex ante* probability of making product 2 times expected profitability in product 2 until death, and the **free entry condition** is:

$$v_e = \left[\frac{G\left(\varphi^{**}\right) - G\left(\varphi^{*}\right)}{\delta}\right]\overline{\pi}_1 + \left[\frac{1 - G\left(\varphi^{**}\right)}{\delta}\right]\overline{\pi}_2 = f_e, \tag{16}$$

where $\overline{\pi}_i$ is expected or average firm profitability in product market i. Equilibrium revenue and profit in each market are constant elasticity functions of firm productivity (equation (7)) and, therefore, average revenue and profit are equal respectively to the revenue and profit of a firm with weighted average productivity, $\bar{r}_i = r_i(\widetilde{\varphi}_i)$ and $\overline{\pi}_i = \pi_i(\widetilde{\varphi}_i)$, where weighted average productivity, $\tilde{\varphi}_1(\varphi^*, \varphi^{**})$ and $\tilde{\varphi}_2(\varphi^{**})$, is determined by the *ex post* productivity distributions, $\mu_i\left(\varphi\right)$, and is defined formally in the Appendix.

3.5. Product and Labor Markets

The steady-state equilibrium is characterized by a constant mass of firms entering each period, M_e, and a constant mass of firms producing within each product market, M_i. In steady-state equilibrium, the mass of firms that enter and draw a productivity sufficiently high to produce in a product market must equal the mass of firms already within that product market who die, yielding the following **steady-state stability conditions** (**SC**):

$$\begin{aligned} [1 - G(\varphi^{**})]M_e &= \delta M_2 && (17) \\ [G(\varphi^{**}) - G(\varphi^{*})]M_e &= \delta M_1. && (18) \end{aligned}$$

The firms' equilibrium pricing rule implies that the prices charged for individual varieties are inversely related to firm productivity. The price indices are weighted averages of the prices charged by firms with different productivities, with the weights determined by the *ex post* productivity distributions. Exploiting this property of the price indices, we can write them as functions of the mass of firms producing a product, M_i, and the price charged by a firm with weighted average productivity within each product market, $p_i(\widetilde{\varphi}_i)$:

$$P_1 = M_1^{1/1-\sigma} p_1(\widetilde{\varphi}_1), \qquad P_2 = M_2^{1/1-\sigma} p_2(\widetilde{\varphi}_2) \tag{19}$$

In equilibrium, we also require that the demand for labor used in production, L^p, and entry, L^e, equals the economy's supply of labor, L:

$$L_p + L_e = L. \tag{20}$$

4. Industry Equilibrium

In this section, we characterize general equilibrium which is referenced by the sextuple $\{\varphi^*,\ \varphi^{**},\ P_1,\ P_2,\ R_1,\ R_2\}$, in terms of which all other endogenous variables may be written. In Section 5, we analyze the properties of industry equilibrium and the implications of the endogenous sorting of firms across products for measured productivity. In Section 6, we investigate the implications of self-selection by firms into products for the evaluation of the impact of a policy reform such as industry deregulation, modelled here as a reduction in the sunk costs of entry into the industry.

The equilibrium vector is determined by the following equilibrium conditions: the zero-profit productivity cutoff (equation (13)), the product-indifference productivity cutoff (equation (14)), free entry (16), steady-state stability ((17) and (18)), the values for the equilibrium price indices implied by consumer and producer equilibrium (equation (19)), and labor market clearing (20).

4.1. Relative Supply and Relative Prices

The zero-profit productivity cutoff implies that the revenue of a product 1 firm with productivity φ^* is proportional to the product 1 fixed production cost (equation (13)), while the product-indifference productivity cutoff establishes a relationship between relative revenue in the two markets at productivity φ^{**} and the fixed costs of producing the two products (equation (14)). Profit maximization implies that the relative revenues of two firms making different products depend solely on their relative productivities, relative expenditure shares on the two products, relative price indices, and relative variable costs of production (equation (8)).

Combining these three equations, we obtain a downward-sloping (supply-side) relationship between two key variables: the relative value of the two productivity cutoffs, φ^{**}/φ^*, and the relative price of the two products, $\mathcal{P}$,

$$\frac{\varphi^{**}}{\varphi^*} \equiv \Lambda = \left[\frac{\left(\frac{f_2}{f_1} - 1\right)}{\left[\left(\frac{1-a}{a}\right)^{\psi}\left(\frac{1}{b}\right)^{\sigma-1}\mathcal{P}^{\sigma-\psi} - 1\right]} \right]^{1/(\sigma-1)}. \tag{21}$$

where this relationship is derived under the assumption that products are asymmetric, so that $f_2 > f_1$ and equation (11) holds. In the special case of symmetric products discussed above, where $a = 1 - a$, $\psi = \sigma$, $f_1 = f_2$ and $b_1 = b_2$, our model collapses to the standard model of industry equilibrium. In this case, there is a single cutoff for productivity, the zero-profit cutoff φ^*, and firms who draw a productivity above φ^* are indifferent between the two identical products and therefore manufacture a variety of either product.

Equation (21) is the mathematical statement of the relationship between the two productivity cutoffs captured graphically in Figure 1. As φ^{**} rises relative to φ^*, the fraction of firms producing product 2 falls, and the fraction of firms producing product 1 increases. Equation (21) therefore yields the following intuitive comparative statics. A higher value for the relative price, $\mathcal{P}$, increases profitability in product 2 relative to product 1 and causes the relative number of firms producing product 2 to rise, i.e. a reduction in φ^{**} relative to φ^{*}, since $\sigma > \psi$. For a given value for the relative price, $\mathcal{P}$, a higher fixed cost for product 2 , f_2, reduces profitability in product 2 and causes the relative number of firms producing product 2 to fall, i.e. an increase in φ^{**} relative to φ^{*}.

4.2. Relative Demand and Relative Prices

The expressions for the two price indices yield an equation for relative prices as a function of the relative mass of firms and the relative price charged by a firm with weighted average productivity in each product market (equation (19)). The two steady-state stability conditions yield an equation for the relative mass of firms as a function of the two productivity cutoffs (equations (17) and (18)).

Combining these expressions yields an upward-sloping demand-side relationship between the relative value of the two productivity cutoffs and the relative price of the two products:

$$\Psi\left(\frac{\varphi^{**}}{\varphi^{*}}\right) \equiv \left[\frac{b^{\sigma-1}\int_{\varphi^{*}}^{\varphi^{**}} \varphi^{\sigma-1} g\left(\varphi\right) d\varphi}{\int_{\varphi^{**}}^{\infty} \varphi^{\sigma-1} g\left(\varphi\right) d\varphi}\right] = \mathcal{P}^{\sigma-1}. \tag{22}$$

An increase in the relative consumer price index for product 2, $\mathcal{P}$, reduces demand for product 2 relative to product 1 and shrinks the range of productivities where product 2 is produced relative to the range where product 1 is produced, i.e. an increase in φ^{**}/φ^{*}. For a given value of φ^{**}/φ^{*}, an increase in b, the relative variable cost for product 2, raises the price of product 2 varieties relative to product 1 varieties, i.e. an increase in $\mathcal{P}$.

4.3. Free Entry

The free entry condition can be written in a more convenient form using the expression for the zero-profit productivity cutoff, the relationship between the revenues of firms producing varieties in the same market with different productivities, and the supply-side relationship between the two productivity cutoffs derived above. Combining equation (13), $r_i\left(\varphi''\right) = \left(\varphi''/\varphi'\right)^{\sigma-1} r_i\left(\varphi'\right)$, and equation (21), we can write the free entry condition as:

$$v_e = \frac{f_1}{\delta} \int_{\varphi^*}^{\Lambda\varphi^*} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] g(\varphi) d\varphi \qquad (23)$$
$$+ \frac{f_1}{\delta} \int_{\Lambda\varphi^*}^{\infty} \left[\left(\frac{1-a}{a} \right)^{\psi} \left(\frac{1}{b} \right)^{\sigma-1} \mathcal{P}^{\sigma-\psi} \left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} - \frac{f_2}{f_1} \right] g(\varphi) d\varphi = f_e.$$

where Λ is defined in equation (21).

This way of writing the free entry condition clarifies the relationship between the sunk cost of entry and the zero-profit productivity cutoff. An increase in the sunk entry cost, f_e, requires an increase in the expected value of entry, v_e. Since the expected value of entry above is monotonically decreasing in φ^*, this requires a fall in the zero-profit productivity cutoff. Intuitively, the higher sunk cost of entering the industry reduces the mass of entrants, which increases *ex post* profitability, enabling lower productivity firms to cover their fixed production costs and survive in the industry.

4.4. Steady-state Stability, Labor Market Clearing and Goods Market Clearing

Using the steady-state stability conditions to substitute for the *ex ante* probability of producing each product in the free entry condition, total payments to labor used in entry equal total industry profits: $L_e = M_e f_e = M_1 \bar{\pi}_1 + M_2 \bar{\pi}_2 = \Pi$ (by choice of numeraire, $w = 1$). The existence of a competitive fringe of potential entrants means that firms enter until the expected value of entry equals the sunk entry cost, and as a result the entire value of industry profits is paid to labor used in entry.

Total payments to labor used in production equal the difference between industry revenue, R, and industry profits, Π: $L_p = R - \Pi$. Taking these two results together, total payments to labor used in both entry and production equal industry revenue, $L = R$. Substituting for R in the expressions for L_e and L_p above, this establishes that the labor market clears.

In equilibrium we also require the goods market to clear, which implies that the value of expenditure equals the value of revenue for each product. Utility maximization implies that the consumer allocates the expenditure shares $\alpha_1(\mathcal{P})$ and $(1 - \alpha_1(\mathcal{P}))$ to the two products. Imposing expenditure equals revenue for each product, goods market clearing may be expressed as:

$$R_1 = \alpha_1(\mathcal{P}) R, \qquad R_2 = (1 - \alpha(\mathcal{P})) R. \qquad (24)$$

4.5. Existence and Uniqueness of Equilibrium

Proposition 1 *There exists a unique value of the equilibrium vector* $\{\varphi^*, \varphi^{**}, P_1, P_2, R_1, R_2\}$*. All other endogenous variables of the model may be written as functions of this equilibrium vector.*

Proof. See Appendix. ■

Combining the supply-side relationship between the relative productivity cutoffs and relative prices in equation (21) with the demand-side relationship in equation (22) yields a unique equilibrium value of φ^{**}/φ^{*} and $\mathcal{P} = P_2/P_1$. In the proof of Proposition 1, we establish that at the unique equilibrium value of $\mathcal{P}$, $\varphi^{*} > 0$ and $\varphi^{**} > \varphi^{*}$, so that both products are produced in equilibrium.

5. Properties of Industry Equilibrium

In this section, we first discuss the central prediction of the model that firms endogenously sort across products depending on their heterogeneous characteristics. We next examine the implications of this result for measured productivity.

5.1. Endogenous Sorting of Firms Across Products

Proposition 2 *There is endogenous sorting of firms across products such that higher productivity firms choose the higher fixed cost product.*

Proof. See Appendix. ■

As shown in Figure 1, lower productivity firms choose to manufacture the low fixed cost product 1, while higher productivity firms self-select into the higher fixed cost product 2. The intuition for this result is that higher productivity firms have lower marginal costs, and so charge lower equilibrium prices, and therefore sell a larger equilibrium quantity of output. Higher productivity firms are thus able to spread the higher fixed cost of product 2 over a larger number of units of output.

The productivity thresholds $\{\varphi^{*}, \varphi^{**}\}$ that determine the range of productivities where products and 2 are manufactured depend on both the parameters of the production technology $\{f_1, f_2\ , b\}$ and those of demand $\{a, \psi, \sigma\}$. Intuitively, technology and demand parameters each influence the slope of the profit functions in Figure 1, and so influence φ^{*} and φ^{**}. The roles of technology and demand can be seen particularly clearly for the case of a Pareto productivity distribution. In this case, the expression for relative demand in equation (22) simplifies, and combining relative demand and relative supply, the ratio of the two productivity cutoffs $\varphi^{**}/\varphi^{*} > 1$ is implicitly defined as follows:

$$\left[\left(\frac{\varphi^{**}}{\varphi^{*}}\right)^{\gamma} - 1\right]^{\frac{1}{\sigma-1}} = \left[\left(\frac{\varphi^{**}}{\varphi^{*}}\right)^{1-\sigma}\left(\frac{f_2}{f_1} - 1\right) + 1\right]^{\frac{1}{\sigma-\psi}}\left(\frac{a}{1-a}\right)^{\frac{\psi}{\sigma-\psi}} b^{\frac{\psi-1}{\sigma-\psi}}. \tag{25}$$

where $\gamma \equiv z - \sigma + 1$ and z is the shape parameter of the Pareto distribution

Taking the technology parameters in this expression first, an increase in the fixed cost of production for product 2 relative to product 1 (f_2/f_1) increases φ^{**} relative to φ^*. Intuitively, as the fixed cost for product 2 rises relative to that for product 1, a higher productivity is required for a firm to charge a low enough price and generate enough revenue to cover the higher fixed cost for product 2. Similarly, an increase in the variable cost of production for product 2 relative to product 1 (b) increases φ^{**} relative to φ^*.

Taking the demand parameters in the expression next, an increase in the weight of product 1 in consumer utility (a) increases φ^{**} relative to φ^*. The reason is that, as consumers increase the share of expenditure on product 1 and reduce that on product 2, a higher productivity is required for a firm to charge a low enough price and generate enough revenue to cover the higher fixed cost for product 2. Finally, the elasticities of substitution σ and ψ determine the impact of marginal cost differences on relative revenue, and hence also influence the relative range of productivities where products 1 and 2 are produced.

5.2. Implications for Measured Firm Productivity

We now examine the implications of the endogenous sorting of firms across products for measured productivity. We follow a large theoretical and empirical literature in defining measured productivity as the residual from the estimation of a production function (see Marschak and Andrews 1944, Olley and Pakes 1996 and Levinsohn and Petrin 2003 among many others).[7] While our model assumes a single factor of production, and so labor productivity and total factor productivity (TFP) coincide, the bias in measured productivity that we establish in this section will continue to apply in models with additional factors of production. Indeed, expanding the number of factors of production typically increases the number of parameters in the production technology that can vary across products within industries, and so introduces additional possible sources of bias. With labor as the sole factor of production in equation (5), the production technology for a particular product i can be rewritten in the following form:

$$\ln q = \ln b_i + \ln (l - f_i) + \ln \varphi \tag{26}$$

where $\ln \varphi$ is true firm productivity.

The central problem in measuring productivity within our framework is that firms make endogenous decisions about which product to manufacture at a more disaggregated level than observed in the data. Neither product choice nor the true productivity of firms is directly observed. In order to measure productivity, the researcher is obliged to estimate a single production function for the industry as a whole. But

[7] The main alternative approach is to use index number measures of productivity following Caves *et al.* (1982a,b). We focus in the discussion here on the production function estimation approach because the standard index number measures assume constant returns to scale whereas our model features increasing returns to scale. Nonetheless, the endogenous sorting of firms across products introduces analogous biases into index number measures of productivity.

because there is endogenous sorting of firms across products based on firm productivity, variation in true firm productivity cannot be distinguished from variation in the parameters of the production technology across products. The variation in measured firm productivity reflects both true differences in firm productivity and firms' non-random decisions about which products to manufacture.

The resulting bias in measured productivity can be characterized and compared to the other sources of bias in measured productivity using the structure of our model. Taking expectations in the production technology (26) conditional on observed factor inputs yields:

$$E\left[\ln q|\ln l\right]=\underbrace{\left[1-G\left(\varphi^{*}\right)\right]}_{\text{Term }A}\left[\begin{array}{c}\underbrace{\left(\frac{G\left(\varphi^{**}\right)-G\left(\varphi^{*}\right)}{1-G\left(\varphi^{*}\right)}\right)}_{\text{Term }C}\underbrace{E\left[\ln q_{1}|\ln l\right]}_{\text{Term }D}\\+\underbrace{\left(\frac{1-G\left(\varphi^{**}\right)}{1-G\left(\varphi^{*}\right)}\right)}_{\text{Term }E}\underbrace{E\left[\ln q_{2}|\ln l\right]}_{\text{Term }F}\end{array}\right], \tag{27}$$

$$\text{where}\qquad \begin{array}{c}E\left[\ln q_{1}|\ln l\right]=\left[\ln b_{1}+\ln\left(l-f_{1}\right)+E\left[\ln\varphi|\ln l\right]\right]\\E\left[\ln q_{2}|\ln l\right]=\left[\ln b_{2}+\ln\left(l-f_{2}\right)+E\left[\ln\varphi|\ln l\right]\right]\end{array}; \tag{28}$$

where $i\in\{1,\ 2\}$ indexes firm product choice; Term A captures the exit selection problem; Terms C and E capture the product selection problem; and Terms D and F incorporate the endogeneity and specification problems.

Now suppose that a researcher ignores the endogenous sorting of firms across products and posits a production technology that takes the same form as equation (26) but imposes common parameters across all firms within the industry $\{\bar{b},\ \bar{f}\}$. Taking expectations in the production technology conditional on observed factor inputs yields:

$$E\left[\ln q|\ln l\right]=\underbrace{\left[1-G\left(\varphi^{*}\right)\right]}_{\text{Term }A}\underbrace{\left[\ln\bar{b}+\ln\left(l-\bar{f}\right)+E\left[\ln u|\ln l\right]\right]}_{\text{Term }G}, \tag{29}$$

where $\ln u$ is measured firm productivity; Term A captures the exit selection problem; and Term G incorporates the product selection, endogeneity and specification problems.

Comparing equations (27), (28) and (29), the expected values of measured and true firm productivity are related to one another as follows:

$$\begin{aligned}E\left[\ln u|\ln l\right]\ =\ &\left(\frac{G\left(\varphi^{**}\right)-G\left(\varphi^{*}\right)}{1-G\left(\varphi^{*}\right)}\right)\ln b_{1}+\left(\frac{1-G\left(\varphi^{**}\right)}{1-G\left(\varphi^{*}\right)}\right)\ln b_{2}-\ln\bar{b}\\&+\left(\frac{G\left(\varphi^{**}\right)-G\left(\varphi^{*}\right)}{1-G\left(\varphi^{*}\right)}\right)\ln\left(l-f_{1}\right)+\left(\frac{1-G\left(\varphi^{**}\right)}{1-G\left(\varphi^{*}\right)}\right)\ln\left(l-f_{2}\right)\\&-\ln\left(l-\bar{f}\right)+E\left[\ln\varphi|\ln l\right]\end{aligned} \tag{30}$$

where the endogenous sorting of firms across products implies that the error term u in equation (29) is correlated with factor input choices $\ln\left(l-\bar{f}\right)$, since u itself includes terms in factor inputs.

Note that the bias in measured productivity that is caused by the endogenous sorting of firms across products is not merely due to the heterogeneity in parameters across products within industries (an aggregation problem), but also due to the systematic correlation between product choice and true productivity (a selection problem). In equation (30), the weights on the parameters of the production technology that reflect the probability of manufacturing each product are themselves functions of true firm productivity φ.

While our discussion here has largely abstracted from the other sources of bias in productivity measurement, they can clearly be introduced into the analysis, and the bias in measured productivity as a result of firms' self-selection into products remains. Furthermore, if other sources of bias in productivity measurement are introduced, they interact with the endogenous sorting of firms across products. For example, the standard endogeneity problem in production function estimation, examined in Olley and Pakes (1996) and Levinsohn and Petrin (2003), arises because higher firm productivity raises labor's marginal product and so increases employment ($E\left[\ln\varphi|\ln l\right]\neq 0$). The endogenous self-selection of firms into products within our model magnifies this endogeneity problem. If higher productivity leads a firm to choose product 2 instead of product 1, there is a further rise in the marginal productivity of labor due to the lower variable cost of product 2, which enhances the increase in employment.

Using our framework, it is possible to sign the direction of the bias in measured productivity that results from the endogenous sorting of firms across products. Suppose that the researcher imposes common parameters across all firms within the industry and uses these parameters $\{\bar{b},\bar{f}\}$ to construct measured productivity. Measured and true firm productivity are defined as follows:

$$\bar{u}_i=\frac{\bar{b}q_i}{\left(l_i-\bar{f}\right)},\qquad \varphi_i=\frac{b_iq_i}{(l_i-f_i)}, \tag{31}$$

Taking the ratio of measured and true productivities for a higher productivity firm manufacturing product 2 and a lower productivity firm manufacturing product 1, we obtain:

$$\frac{u_2}{u_1}=\left(\frac{q_2}{q_1}\right)\left(\frac{l_1-\bar{f}}{l_2-\bar{f}}\right),\qquad \frac{\varphi_2}{\varphi_1}=\left(\frac{q_2}{q_1}\right)\left(\frac{b_2}{b_1}\right)\left(\frac{l_1-f_1}{l_2-f_2}\right). \tag{32}$$

Proposition 3 *The percentage difference in measured productivity (u) between a firm manufacturing product 1 and another firm manufacturing product 2 is greater than the percentage difference in true productivity (φ) if* $\left(\frac{b_2}{b_1}\right)\left(\frac{l_1-f_1}{l_2-f_2}\right)<\left(\frac{l_1-\bar{f}}{l_2-\bar{f}}\right)$ *and is less than the percentage difference in true productivity (φ) if* $\left(\frac{b_2}{b_1}\right)\left(\frac{l_1-f_1}{l_2-f_2}\right)>\left(\frac{l_1-\bar{f}}{l_2-\bar{f}}\right)$.

Proof. See Appendix. ■

Both true and measured productivity depend on comparisons of output relative to factor inputs. On the one hand, high productivity firms which manufacture product 2 generate more output relative to their variable factor inputs than low productivity firms which manufacture product 1, not only because of their higher true productivity, but also because they self-select into the product with the lower variable cost of production. The measure of productivity that imposes common parameters across products does not control for the difference in variable cost between the two products, and therefore ascribes all of the higher output relative to variable factor inputs of high productivity firms to their greater productivity. On the other hand, high productivity firms which manufacture product 2 have a higher fixed labor requirement than low productivity firms which manufacture product 1. This higher fixed labor requirement for product 2 reduces output relative to labor input. The measure of productivity that imposes common parameters across products does not control for the difference in fixed cost between the two products, and therefore attributes the lower output relative to labor input caused by the higher fixed cost for product 2 to a lower productivity. Therefore, the difference in measured productivity between a firm manufacturing product 2 can be either greater or less than the difference in true productivity, depending on the size of the difference in the variable and fixed cost parameters, as specified in the inequalities contained in Proposition 3. When the difference in variable cost parameters across products is large relative to the difference in fixed cost parameters, the dispersion in measured productivity will be greater than the dispersion in true productivity.[8]

To gauge the quantitative importance of the bias in productivity measurement introduced by the endogenous sorting of firms across products, we numerically solve the model assuming a Pareto productivity distribution. We first take standard values for the main parameters of the model from the existing heterogeneous firms literature (Bernard *et al.* 2003, Melitz 2003, Ghironi and Melitz 2005 and Bernard *et al.* 2006) as discussed further in the appendix. We next evaluate true and measured productivity dispersion for a variety of degrees of product asymmetry. In the interests of brevity, we report results holding constant the fixed cost parameters $\{f_1, f_2\}$ and the variable cost parameter for product 1 ($b_1 = 1$) and varying the variable cost parameter for product 2 ($b_2 = b$). We assume that the fixed production cost for product 2 is 20% higher than that for product 1 and we consider variable cost parameters for product 2 between 0.8 and 0.5. As shown in Table 2, reductions in b_2 decrease φ^{**}

[8] In the model, the dispersion of firm sizes is determined by the dispersion in marginal costs. As a result, the self-selection of higher productivity firms into lower marginal cost products will magnify inequality in the firm-size distribution. The higher the elasticity of substitution between products, the greater the impact of variation in marginal costs on relative firm sizes. Hence, the endogenous sorting of firms across products will have a larger impact on the firm-size distribution for higher values of the elasticity of substitution.

and increase φ^*. The intuition for these comparative statics is that reductions in b_2 enhance profitability in product 2, which induces firms to switch from product 1 to 2, and so leads to a decrease in φ^{**}. At the same time the enhanced profitability of product 2 raises the expected value of entry, which induces increased entry and stronger product market competition, and so leads to an increase in φ^*.

To evaluate the quantitative implications of product asymmetry for the bias in productivity measurement, the final two columns of Table 2 compare the ratios of true and measured productivity for the least productive firm making product 1 (with productivity φ^*) and the least productive firm making product 2 (with productivity φ^{**}). For each value of b_2, the ratio of measured productivity u^{**}/u^* is greater than the ratio of true productivity φ^{**}/φ^*, indicating that for the parameter values chosen the impact of product 2's lower variable cost in enhancing measured productivity differences dominates the impact of product 2's higher fixed cost in diminishing measured productivity differences. As b_2 falls from 0.8 to 0.5, φ^{**}/φ^* declines from 1.81 to 1.27. In contrast, u^{**}/u^* *rises* from 2.95 to 3.20. The reason is that the evolution of relative measured productivity depends on both relative true productivity and the relative variable costs of production for product 2. Therefore, as b_2 falls, output rises relative to variable factor inputs for product 2, which increases relative measured productivity. Comparing the final two columns of the table, the endogenous sorting of firms across products can cause measured productivity to diverge substantially from true productivity.

Variable Cost Parameter Product 2 (b_2)	φ^*	φ^{**}	Ratio of True Productivity φ^{**}/φ^*	Ratio of Measured Productivity u^{**}/u^*
0.8	0.72	1.81	2.50	2.95
0.7	0.73	1.60	2.20	2.96
0.6	0.74	1.44	1.94	3.05
0.5	0.75	1.27	1.70	3.20

Table 2: Numerical Solution of the Model

Our analysis also suggests approaches to controlling for the bias introduced into productivity measures by the endogenous sorting of firms across products. Clearly, the collection of more complete data where both output and factor inputs are reported by product for individual firms directly addresses the concern. Moreover, the collection of more detailed data on products facilitates the estimation of structural models of industry equilibrium that explicitly take into account the endogenous sorting of firms across products. This approach is in line with an influential body of recent research in industrial organization that seeks to exploit highly disaggregated information on specific product markets, including among others Berry, Levinsohn and Pakes (1995), Goldberg (1995) and Petrin (2002). While conceptually attractive, the data collection requirements of this approach are extremely demanding and are not likely to be satisfied for the entire manufacturing sector of the United States or other countries in the immediate future. Therefore empirical researchers who are concerned with the productivity of the manufacturing sector as a whole, or who wish to examine the macroeconomic effects of policy reforms such as trade liberalization, or who wish to exploit differential variation across industries and over time in policy variables such as tariffs will continue to face the bias introduced into productivity measures by the endogenous sorting of firms across products.

Nonetheless, even in the absence of complete data on output and inputs by firm and by product, the productivity of firms (or plants) can be still estimated consistently if they can be separated into groups that manufacture a single product. In this case, productivity can be estimated across firms (or plants) manufacturing the same product. Since the product is the same across firms, the bias from the endogenous sorting of firms across products is eliminated. This second approach is empirically relevant because some Census of Production datasets, such as that of the United States, do report information on manufacturing activity at a more disaggregated level than the main four-digit industry that has been the focus of much of the empirical work using these datasets. Indeed, section 2 presented empirical evidence of substantial heterogeneity in productivity across single-product plants within four-digit industries, suggesting that going forward there is scope for exploiting the product-level information in these datasets.

There are a number of caveats to each of these approaches. One still needs to control for other sources of bias in productivity measurement such as the exit selection, endogeneity and specification problems. Moreover, focusing on single-product firms (or plants) in the second approach excludes a substantial proportion of manufacturing activity. Finally, many editions or versions of products are released by firms, and products are frequently customized to particular customers. No matter how detailed the data on output and factor inputs, there is always likely to be scope for firms to make decisions about which product to make at a more disaggregated level than observed in the data, in which case the endogenous sorting of firms across products will remain a source of bias in productivity measures.

6. Policy Reforms and Measured Productivity

The self-selection of firms into products is not only a source of bias in measured productivity but also complicates the evaluation of policy reforms such as trade liberalization or industry deregulation. Existing research on such policy reforms emphasizes within-industry reallocation as a source of aggregate productivity gains (see for example Pavcnik 2002, Tybout 2003, Bernard, Eaton, Jensen and Kortum 2003, Melitz 2003, Trefler 2004). Reductions in entry barriers lead to exit by low productivity firms and a reallocation of output towards high productivity firms that raises aggregate industry productivity. Our model suggests a new dimension of reallocation following trade liberalization: the endogenous re-sorting of firms across products. While this new dimension of reallocation also acts as a source of aggregate productivity gains, productivity measures that fail to control for the endogenous sorting of firms across products yield biased estimates of the true impact of deregulation on firm and industry productivity.

In this section, we first use our model to determine the true impact of deregulation on firm and industry productivity before next establishing the biases that result from a failure to control for self-selection of firms into products. To make these points in the clearest way possible, we consider the particularly tractable version of the model discussed above where productivity follows a Pareto distribution.

Proposition 4 *A reduction in entry barriers, f_e, leads to:*
(a) *A rise in the zero profit productivity cutoff, φ^*, and a rise in the product indifference productivity cutoff, φ^{**}*
(b) *A rise in weighted average industry productivity*

Proof. See Appendix. ■

The reduction in entry barriers (a reduction in f_e) increases the two productivity cutoffs (φ^* and φ^{**}), and so raises weighted average productivity for each product ($\tilde{\varphi}_1$ and $\tilde{\varphi}_2$), and hence weighted average productivity for the industry as a whole. With a Pareto productivity distribution, the ratio of the two productivity cutoffs is independent of the sunk costs of entry, as can be seen from equation (25), and so φ^{**} rises by the same proportion as φ^*.

The intuition for these results can be obtained by considering the impact of the reduction in barriers to entry at the initial steady-state equilibrium. As the sunk entry cost falls below the expected value of entry, a larger mass of firms, M_e, enters the industry. For given values of φ^* and φ^{**}, a larger mass of entrants implies a larger mass of firms with productivity realizations high enough to manufacture each product, which increases product market competition and reduces *ex post* profitability for each product.

The reduction in *ex post* profitability means that some low productivity firms are now no longer able to cover the fixed costs of producing product 1. Hence, in equilibrium the zero-profit productivity cutoff φ^* rises. As φ^* rises for a given value of φ^{**}, this reduces the mass of firms in product 1 relative to the mass of firms in product 2, thereby increasing product 1's relative profitability. Hence, some higher productivity firms that previously made product 2 now find it more profitable to produce the low fixed cost product 1 and φ^{**} also rises.

The rise in φ^* and φ^{**} implies that some low productivity firms that previously made product 1 exit, while some higher productivity firms that previously made product 2 switch to product 1. For both these reasons, weighted average productivity in product 1, $\widetilde{\varphi}_1$, will rise. Since the firms that switch from product 2 to product 1 are of lower productivity than those who continue to make product 2, weighted average productivity in product 2, $\widetilde{\varphi}_2$, will also rise. Therefore, the endogenous resorting of firms across products following industry deregulation provides a new source of reallocation in addition to the entry and exit of firms, which contributes towards the increase in average industry productivity. As well as the rise in the threshold productivity below which firms exit (φ^*), there is a rise in the threshold productivity at which product 2 is manufactured (φ^{**})

The failure to control for the endogenous sorting of firms across products, however, implies that measured productivity provides a biased estimate of the impact of industry deregulation on true firm and industry productivity. True firm productivity is a parameter, which is drawn upon entry, and is therefore unaffected by industry deregulation. But measured productivity depends upon which product is chosen, and so firms that change their product in response to industry deregulation experience changes in measured productivity even though true firm productivity remains constant. Since industry deregulation leads to a rise in φ^{**}, there is a range of productivities where firms previously manufactured product 2 and now manufacture product 1 following industry deregulation. If the increase in output relative to factor inputs due to a lower fixed cost of product 1 exceeds the reduction due to a higher variable cost, the measured productivity of a firm in this range of productivities will rise, whereas if the converse is true, the measured productivity of the firm will fall.[9]

To examine the aggregate impact of industry deregulation, a measure of weighted average productivity can be constructed, which is directly analogous to the true value of weighted average productivity, but which aggregates the measured productivity of individual firms. The bias in measured firm productivity introduced by the endogenous sorting of firms across products distorts measured weighted average productivity, so that the change in measured weighted average productivity provides a biased estimate of the impact of industry deregulation on true weighted average productivity.

In short, the endogenous sorting of firms across products not only introduces a bias into measures of productivity, but also provides a new source of reallocation that complicates the evaluation of the productivity effects of policy reforms such as industry deregulation.

[9] Although for simplicity true firm productivity is modelled here as a parameter, the increase in product market competition following industry deregulation could lead in a richer framework to an increase in true firm productivity. In this case, for firms in the range of productivities where the low variable cost product 2 was previously manufactured and the high variable cost product 1 is now manufactured, the change measured firm productivity would again yield a biased estimate of the change in true firm productivity following industry deregulation.

7. Conclusions

Firms make their production decisions at a level of aggregation that is more detailed than the one typically available to empirical researchers. We demonstrate that the endogenous sorting of firms into products introduces a new source of bias into standard measures of productivity that augments the conventional biases associated with exit self-selection, endogeneity and mis-specification of the functional form of the production technology or market structure. We show that standard measures of productivity differences across firms reflect both true differences in productivity and firms' non-random decisions about which products to manufacture.

To explore the bias in measured productivity caused by the endogenous sorting of firms across products, the paper develops a theoretical model that is a natural extension of existing models of industry dynamics to allow firms to decide which product to manufacture within an industry. If a researcher ignores the endogenous sorting of firms across products and posits a single production technology for the industry as a whole, variation in true firm productivity cannot be distinguished from variation in the parameters of the production technology across products. In our framework, higher productivity firms self-select into the product with the higher fixed cost and lower variable cost product. Therefore, if the increase in output relative to factor inputs due to a lower variable cost exceeds the reduction due to a higher fixed cost, the difference in measured productivity between firms specializing in distinct products is larger than the difference in true productivity, whereas otherwise the converse is true. We use the structure of the model to determine firms' product choice and the direction and magnitude of the bias in measured productivity as a function of the fixed and variable cost parameters.

Our analysis suggests a number of areas for further inquiry. The collection of more highly disaggregated data for specific product markets and the estimation of structural models of industry equilibrium that explicitly take into account the endogenous sorting of firms across products are promising areas of active research. More generally, the development of Census of Production datasets that contain information on not only the main industry of a firm but also all the industries and products where a firm is active is an area of priority. Where such information exists, there is evidence of heterogeneity in productivity across plants specializing in different products within industries. More generally, further study of a firm's decision about which product to manufacture at a production facility can enhance our understanding of the extensive variation in productivity across firms within industries as well as the impact of policy reforms such as industry deregulation on both firm and industry productivity.

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A Appendix: Multi-product Firms

In our baseline analysis, we assume that managerial diseconomies of scope are such that a firm can only manufacture one product. We now relax this assumption to allow firms to potentially manufacture both products.

The determination of industry equilibrium follows a very similar line of reasoning as in the main text. Low productivity firms are only able to generate enough revenue to cover fixed costs for the low fixed cost product 1, and so manufacture one product. High productivity firms are able to enough revenue to cover fixed costs for both the low fixed cost product 1 and the high fixed cost product 2, and so manufacture both products.

The expressions for weighted average productivity and the price indices are modified to take into account the fact that high productivity firms now manufacture product 1 as well as product 2. The free entry condition is also modified. The expected value of entry is now equal to the *ex ante* probability of becoming a single-product firm times the expected profitability of a single-product firm manufacturing product 1 until death, plus the *ex ante* probability of becoming a multi-product firm times the expected profitability of a multi-product firm in the two products together until death:

$$v_e = \frac{[G(\varphi^{**}) - G(\varphi^{*})]}{\delta}\bar{\pi}_1^{SP} + \frac{[1 - G(\varphi^{**})]}{\delta}\left[\bar{\pi}_1^{MP} + \bar{\pi}_2^{MP}\right] \tag{33}$$

If product choice cannot be observed, and hence single product firms cannot be distinguished from multi-product firms, the endogenous sorting of firms continues to introduce a bias into measured productivity. If a researcher posits a single production technology for the industry as a whole, variation in true firm productivity cannot be distinguished from variation in the parameters of the production technology across products. Low productivity firms who manufacture product 1 have high variable costs, while the variable costs for high productivity firms who manufacture both products are a weighted average of the high variable costs for product 1 and the low variable costs for product 2.

The existence of multi-product firms also introduces other considerations. If output is measured using revenue rather than quantity data, the accurate measurement of productivity requires product-specific price deflators and the ability to control for the endogenous composition of firm output across products (see for example the discussion in Levinsohn and Melitz 2006 and De Loecker 2005). More generally, with multi-product firms, a firm decides not only which product to manufacture but also the number of products to manufacture and the composition of firm output across those products.

B Appendix: Theoretical Derivations

B1. Weighted Average Productivity and Average Profitability

$$\begin{aligned}
\widetilde{\varphi}_1(\varphi^*, \varphi^{**}) &= \left[\frac{1}{G(\varphi^{**}) - G(\varphi^*)} \int_{\varphi^*}^{\varphi^{**}} \varphi^{\sigma-1} g(\varphi)\, d\varphi\right]^{1/(\sigma-1)} \\
\widetilde{\varphi}_2(\varphi^{**}) &= \left[\frac{1}{1 - G(\varphi^{**})} \int_{\varphi^{**}}^{\infty} \varphi^{\sigma-1} g(\varphi)\, d\varphi\right]^{1/(\sigma-1)}
\end{aligned} \tag{34}$$

Using the relationship between the revenues of firms producing varieties in the same and in different markets, as well as the expression for the zero-profit productivity cutoff and the CES expenditure share, average profit in the two product markets, $\bar{\pi}_i = \pi_i(\tilde{\varphi}_i)$ may be written as follows:

$$\overline{\pi}_1(\varphi^*, \varphi^{**}) = \left[\left(\frac{\widetilde{\varphi}_1(\cdot)}{\varphi^*}\right)^{\sigma-1} - 1\right] f_1 \tag{35}$$

$$\overline{\pi}_2(\varphi^*, \varphi^{**}, \mathcal{P}) = \left[\left(\frac{1-a}{a}\right)^{\psi} \left(\frac{1}{b}\frac{\widetilde{\varphi}_2(\cdot)}{\varphi^*}\right)^{\sigma-1} \mathcal{P}^{\sigma-\psi} - \frac{f_2}{f_1}\right] f_1 \tag{36}$$

B2. Proof of Proposition 1

Proof. We begin by determining the equilibrium sextuple: $\{\varphi^*, \varphi^{**}, P_1, P_2, R_1, R_2\}$. First, we use the relative supply and relative demand relationships in equations (21) and (22) to establish that there exist unique equilibrium values of φ^{**}/φ^* and $\mathcal{P}$. Rearranging the product supply relationship, we obtain:

$$\mathcal{P} = b^{\frac{\sigma-1}{\sigma-\psi}} \left(\frac{a}{1-a}\right)^{\frac{\psi}{\sigma-\psi}} \left[\left(\frac{\varphi^{**}}{\varphi^*}\right)^{1-\sigma} \left(\frac{f_2}{f_1} - 1\right) + 1\right]^{\frac{1}{\sigma-\psi}}. \tag{37}$$

Since $\sigma > 1$, the right-hand side is monotonically decreasing in φ^{**}/φ^* and is graphed in $(\mathcal{P}, \varphi^{**}/\varphi^*)$ space in Figure 2. $\mathcal{P}$ takes the value $(f_2/f_1)^{1/(\sigma-\psi)}\,(a/(1-a))^{\psi/(\sigma-\psi)}\, b^{(\sigma-1)/(\sigma-\psi)} > 0$ at $\varphi^{**}/\varphi^* = 1$ and converges to a lower value of $(a/(1-a))^{\psi/(\sigma-\psi)}\, b^{(\sigma-1)/(\sigma-\psi)} > 0$ as φ^{**}/φ^* tends to infinity.

Turning now to the product demand relationship (equation (22)), the left-hand side is monotonically increasing in φ^{**}/φ^* and is also graphed in $(\mathcal{P}, \varphi^{**}/\varphi^*)$ space below. As φ^{**}/φ^* approaches 1, $\mathcal{P}$ converges to 0. As φ^{**}/φ^* tends to infinity, $\mathcal{P}$ converges to ∞.

Therefore, as shown in 2, there exists a unique equilibrium value of ($\mathcal{P}$, φ^{**}/φ^*) where both the relative supply and relative demand relationships are satisfied and

where $\varphi^{**}/\varphi^* > 1$.
Given values of $\Lambda \equiv \varphi^{**}/\varphi^*$ and $\mathcal{P}$, equation (23) is monotonically decreasing in φ^*:

$$\frac{dv_e}{d\varphi^*} < 0 \tag{38}$$

$$\Leftrightarrow \quad \underbrace{\frac{f_1}{\delta}\int_{\varphi^*}^{\Lambda\varphi^*} \varphi^{\sigma-1}(1-\sigma)(\varphi^*)^{-\sigma}g(\varphi)d\varphi}_{\text{Term A}} + \underbrace{\frac{f_1}{\delta}\Lambda\left[\Lambda^{\sigma-1}-1\right]g(\Lambda\varphi^*)}_{\text{Term B}}$$

$$+\underbrace{\frac{f_1}{\delta}\int_{\Lambda\varphi^*}^{\infty}\left(\frac{1-a}{a}\right)^{\psi}\left(\frac{1}{b}\right)^{\sigma-1}\mathcal{P}^{\sigma-\psi}\varphi^{\sigma-1}(1-\sigma)(\varphi^*)^{-\sigma}g(\varphi)d\varphi}_{\text{Term C}}$$

$$\underbrace{-\frac{f_1}{\delta}\Lambda\left[\left(\frac{1-a}{a}\right)^{\psi}\left(\frac{1}{b}\right)^{\sigma-1}\mathcal{P}^{\sigma-\psi}\Lambda^{\sigma-1}-\frac{f_2}{f_1}\right]g(\Lambda\varphi^*)}_{\text{Term D}} < 0$$

The sum of Terms B and D may be written as,

$$\frac{f_1}{\delta}\Lambda g(\Lambda\varphi^*)\left[\left(\frac{f_2}{f_1}-1\right)-\Lambda^{\sigma-1}\left(\left(\frac{1-a}{a}\right)^{\psi}\left(\frac{1}{b}\right)^{\sigma-1}\mathcal{P}^{\sigma-\psi}-1\right)\right].$$

where, from the definition of Λ in equation (21), the term in square parentheses is exactly equal to zero. Since $\sigma > 1$, Terms A and C in equation (38) are negative. Hence, $\frac{dv_e}{d\varphi^*} < 0$ for all φ^*. Furthermore, as $\varphi^* \to 0$ in equation (23), $v_e \to \infty$. As $\varphi^* \to \infty$, $v_e \to 0$. Together, equations (21), (22) and (23) determine unique equilibrium values of the three unknowns $(\varphi^*, \varphi^{**}, \mathcal{P})$. Since $\varphi^* > 0$ and $\varphi^{**} > \varphi^*$ both products are indeed produced in equilibrium.
These three elements of the equilibrium vector are sufficient to determine weighted average productivity, $\widetilde{\varphi}_1$ and $\widetilde{\varphi}_2$, in equation (34), as well as average revenue and hence average profitability, $\overline{\pi}_1$ and $\overline{\pi}_2$, in equations (35) and (36).
As shown in the main text, the steady-state stability and free entry conditions (equations (17), (18) and (16)) imply that total revenue, R, is equal to total payments to labor used in both entry and production, L.
Revenue in each product market may be determined from the CES expenditure share (equation (4)) at the equilibrium value of relative prices, $\mathcal{P}$, for which we solved above: $R_1 = \alpha_1(\mathcal{P})L$ and $R_2 = (1-\alpha(\mathcal{P}))L$.
From consumer and producer optimization, the price indices, P_1 and P_2, may be written as functions of the mass of firms, M_1 and M_2, and the price charged by a firm with weighted average productivity, $p_1(\widetilde{\varphi}_1)$ and $p_2(\widetilde{\varphi}_2)$:

$$\begin{aligned} P_1 &= (M_1)^{\frac{1}{1-\sigma}}p_1(\widetilde{\varphi}_1) = \left(\frac{\alpha_1(\mathcal{P})L}{\sigma(\overline{\pi}_1+f_1)}\right)^{\frac{1}{1-\sigma}}\frac{1}{\rho\widetilde{\varphi}_1} \\ P_2 &= (M_2)^{\frac{1}{1-\sigma}}p_2(\widetilde{\varphi}_2) = \left(\frac{(1-\alpha_1(\mathcal{P}))L}{\sigma(\overline{\pi}_2+f_2)}\right)^{\frac{1}{1-\sigma}}\frac{1}{\rho\widetilde{\varphi}_2} \end{aligned}$$

where we have used $M_i = R_i/\overline{r}_i$ and $(\overline{\pi}_1, \overline{\pi}_2, \widetilde{\varphi}_1, \widetilde{\varphi}_2)$ were determined above. We have thus characterized the equilibrium sextuple $\{\varphi^*, \varphi^{**}, P_1, P_2, R_1, R_2\}$.
We now show that all other endogenous variables of the model may be derived from the equilibrium sextuple $\{\varphi^*, \varphi^{**}, P_1, P_2, R_1, R_2\}$.
From equation (19), (M_1, M_2) can be expressed as functions of the price indices (P_1, P_2) and weighted average productivity $(\widetilde{\varphi}_1, \widetilde{\varphi}_2)$ which is determined by $(\varphi^*, \varphi^{**})$ alone. From the analysis in the main text, $M_e = \Pi/f_e = [M_1\bar{\pi}_1 + M_2\bar{\pi}_2]/f_e$, where (M_1, M_2) have just been determined and $(\bar{\pi}_1, \bar{\pi}_2)$ can be derived from $(\varphi^*, \varphi^{**}, \mathcal{P})$.
Total payments to labor used in production in product market i equal the difference between revenue, R_i, and total firm profits, Π_i, in that market. Therefore:

$$\begin{aligned} L_{p1} &= R_1 - \Pi_1 = R_1 - (M_1\bar{\pi}_1) \\ L_{p2} &= R_2 - \Pi_2 = R_2 - (M_2\bar{\pi}_2) \end{aligned}$$

where we have used the choice of labor as numeraire, (R_1, R_2) are part of the equilibrium sextuple, (M_1, M_2) were determined above, and $\overline{\pi}_1$ and $\overline{\pi}_2$ are functions of $(\varphi^*, \varphi^{**}, \mathcal{P})$ alone. Payments to labor used in entry are:

$$L_e = M_e f_e$$

where M_e was determined above.
The first-order conditions for consumer optimization imply:

$$C_1 = R\frac{a^{\psi}P_1^{-\psi}}{\left[a^{\psi}P_1^{1-\psi} + (1-a)^{\psi}P_2^{1-\psi}\right]}, \qquad C_2 = R\frac{(1-a)^{\psi}P_2^{-\psi}}{\left[a^{\psi}P_1^{1-\psi} + (1-a)^{\psi}P_2^{1-\psi}\right]}$$

where $R = L$ and (P_1, P_2) are part of the equilibrium sextuple. ■

B3. Proof of Proposition 2

Proof. This proposition follows immediately from the Proof of Proposition 1 where we have established that $\varphi^* > 0$ and $\varphi^{**} > \varphi^*$. ■

B4. Proof of Proposition 3

Proof. This proposition follows immediately from equation (32) where $u_2/u_1 > \varphi_2/\varphi_1$ if $\left(\frac{b_2}{b_1}\right)\left(\frac{l_1-f_1}{l_2-f_2}\right) < \left(\frac{l_1-\bar{f}}{l_2-\bar{f}}\right)$ and $u_2/u_1 < \varphi_2/\varphi_1$ if $\left(\frac{b_2}{b_1}\right)\left(\frac{l_1-f_1}{l_2-f_2}\right) > \left(\frac{l_1-\bar{f}}{l_2-\bar{f}}\right)$. ■

B5. Proof of Proposition 4

Proof. (a) The expected value of entry in (23) is monotonically decreasing in the zero-profit productivity cutoff φ^*. Therefore, as barriers to entry are reduced (a reduction in the sunk costs of entry f_e), the zero-profit productivity cutoff φ^* must rise so as to reduce the expected value of entry equal to the new lower sunk cost. With a Pareto productivity distribution Λ is unchanged after the fall in the sunk cost of entry from equation (25). Therefore, since $\varphi^{**} = \Lambda\varphi^*$, φ^{**} will rise by the same proportion as $\varphi^* : d\varphi^* > 0$, $d\varphi^{**} > 0$ and $d(\varphi^{**}/\varphi^*) = 0$.

(b) True weighted average productivity in each product market, $\widetilde{\varphi}_1$ and $\widetilde{\varphi}_2$, is defined in equations (34). If productivity is Pareto distributed, $g(\varphi) = zk^z\varphi^{-(z+1)}$ where $k > 0$ and $z > 0$, it follows that $\varphi^{\sigma-1}g(\varphi) = \xi h(\varphi)$ where $h(\varphi) = \gamma k^\gamma \varphi^{-(\gamma+1)}$, $\xi \equiv zk^{z-\gamma}/\gamma$ and $\gamma \equiv z-\sigma+1$. That is, $h(\varphi)$ is a Pareto distribution with minimum productivity k and skewness parameter $\gamma \equiv z-\sigma+1$, where we assume $z > \sigma - 1$ which corresponds to an assumption that the variance of firm sales in the model is finite. Combining these results, it follows that weighted average productivity in the two product markets may be expressed as:

$$\widetilde{\varphi}_1(\varphi^*, \varphi^{**})^{\sigma-1} = \frac{H(\varphi^{**}) - H(\varphi^*)}{G(\varphi^{**}) - G(\varphi^*)} = \frac{z(\varphi^*)^{\sigma-1}}{\gamma}\left[\frac{1-\Lambda^{-\gamma}}{1-\Lambda^{-z}}\right]. \tag{39}$$

$$\widetilde{\varphi}_2(\varphi^{**})^{\sigma-1} = \frac{1 - H(\varphi^{**})}{1 - G(\varphi^{**})} = \frac{z(\varphi^{**})^{\sigma-1}}{\gamma}. \tag{40}$$

where $H(\varphi)$ is the cumulative distribution function corresponding to $h(\varphi)$.
Since $\sigma > 1$, and using the results for the change in the productivity cutoffs above, $d\widetilde{\varphi}_1 = (d\widetilde{\varphi}_1/d\varphi^*)d\varphi^* > 0$ and $d\widetilde{\varphi}_2 = (d\widetilde{\varphi}_2/d\varphi^{**})d\varphi^{**} > 0$. Since true weighted average productivity rises in each product market individually, true weighted average productivity in the industry as a whole also rises. ■

C Appendix: Numerical Solution

We set the elasticity of substitution between varieties equal to $\sigma = 3.8$, which matches estimates using plant-level U.S. manufacturing data in Bernard *et al.* (2003). The elasticity of substitution between products is set equal to $\psi = 2$, which satisfies $\sigma > \psi$. We assume that productivity follows a Pareto distribution: $g(\varphi) = zk^z\varphi^{-(z+1)}$. The minimum value of productivity is set equal to $k = 0.2$, and the Pareto shape parameter is set equal to $z = 3.4$, which satisfies the requirement for the variance of log productivity to be finite: $z > \sigma - 1$.

Changing the sunk cost of entry, f_e, rescales the mass of firms and, without loss of generality, we set $f_e = 1$. Exit in the model includes both the endogenous decision of firms with low productivity draws to leave the industry and exogenous death due to *force majeure* events. Changes in the probability of exogenous firm death, δ, rescale the mass of entrants relative to the mass of firms and, without loss of generality, we set $\delta = 0.025$.

We set the economy's endowment of labor equal to $L = 1000$. Fixed production costs for product 2 are assumed to be 20% higher than those for product 1: $f_1 = 0.01$

and $f_2 = 0.012$. The variable cost parameter for product 1 is normalized to unity ($b_1 = 1$) and we vary the variable cost parameter for product 2 (b_2) between 0.8 and 0.5.

We solve for the general equilibrium of the model for each value of the variable cost parameter for product 2 and report the value of the zero-profit and product indifference cutoffs (φ^* and φ^{**}), the ratio of true productivity at the two cutoffs (φ^{**}/φ^*) and the ratio of measured productivity at the two cutoffs (u^{**}/u^*) when a researcher assumes a common value of the fixed and variable cost parameters for the two products. When evaluating measured productivity, we set the common value of the fixed and variable cost parameters equal to their expected values conditional on successful entry: $\bar{f} = \left(\frac{G(\varphi^{**})-G(\varphi^*)}{1-G(\varphi^*)}\right) f_1 + \left(\frac{1-G(\varphi^{**})}{1-G(\varphi^*)}\right) f_2$ and $\bar{b} = \left(\frac{G(\varphi^{**})-G(\varphi^*)}{1-G(\varphi^*)}\right) b_1 + \left(\frac{1-G(\varphi^{**})}{1-G(\varphi^*)}\right) b_2$.

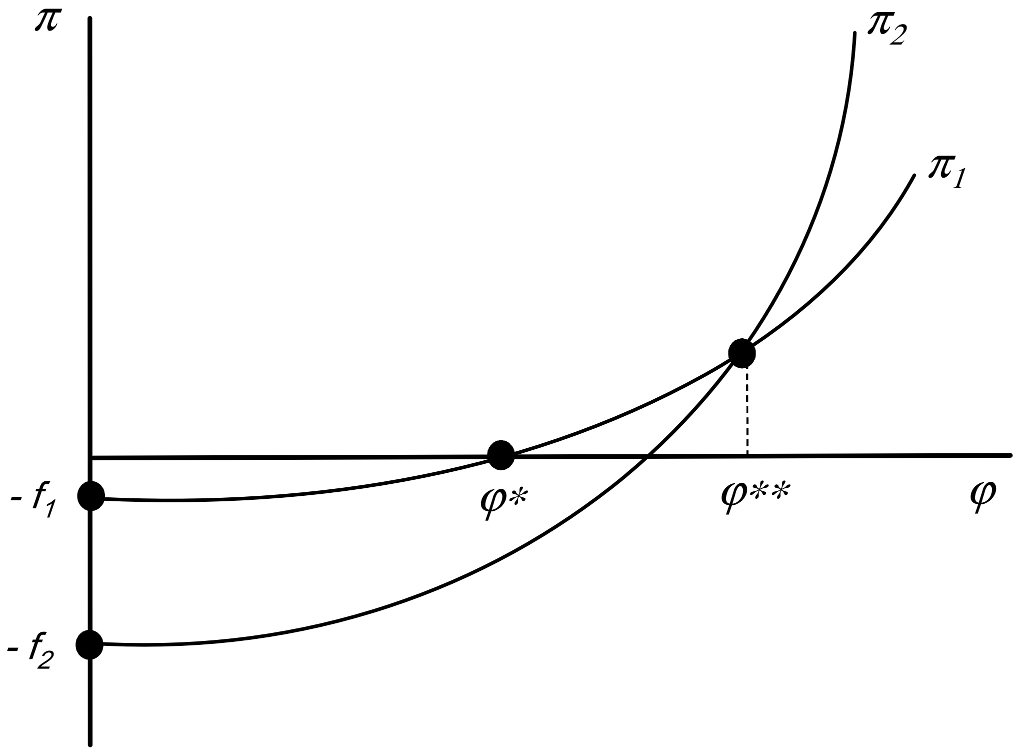

Figure 1: Profit versus Productivity for the Two Products

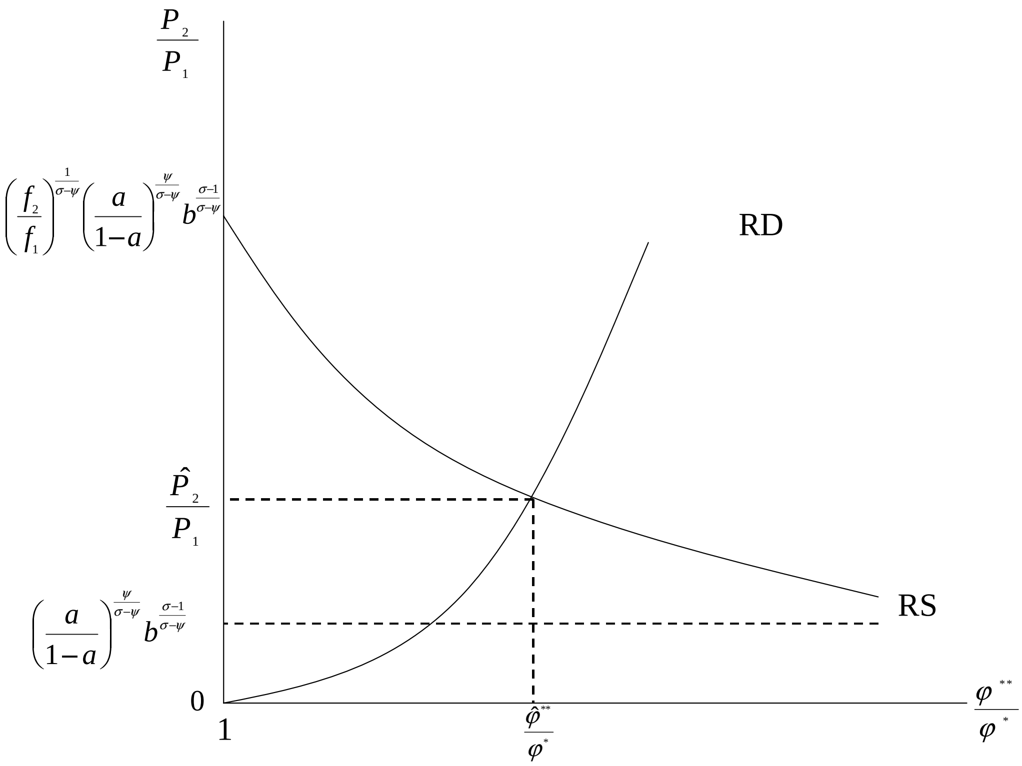

Figure 2: Equilibrium $\mathcal{P}$ and φ^{**}/φ^{*}