

**Slow to Hire, Quick to Fire: Employment Dynamics with
Asymmetric Responses to News***

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Abstract

Concave hiring rules imply that firms respond more to bad shocks than to good shocks. They provide a united explanation for several seemingly unrelated facts about employment growth in macro and micro data. In particular, they generate countercyclical movement in both aggregate conditional “macro” volatility and cross-sectional “micro” volatility as well as negative skewness in the cross section and in the time series at different level of aggregation. Concave establishment level responses of employment growth to TFP shocks estimated from Census data induce significant skewness, movements in volatility and amplification of bad aggregate shocks.

Keyword: business cycles, time varying volatility, asymmetric adjustment, skewness

JEL Classification: D2, D8, E2, J2

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1 Introduction

A long tradition in macroeconomics studies linear models with shocks that have symmetric distributions and constant variance. However, a growing body of micro and macro evidence is inconsistent with this approach. One prominent stylized fact is the countercyclical movement of cross sectional volatility. For example, Figure 1 shows that the dispersion of employment growth rates across firms – here measured by the interquartile range – spikes up in recessions and declines in booms. The figure also illustrates a second familiar example: the negative skewness of aggregate growth rates.¹ We show below that negative skewness also obtains at the firm level, both in the time series and the cross section.

The new facts present macroeconomists with two options: either work with exogenous shocks that have skewed distributions and changing variance, or build nonlinear models. Most of the literature has taken the former approach. For example, many recent studies of the cross section of firms start from exogenous volatility or skewness properties of firm level productivity. Moreover, exogenous “volatility shocks” have been used to complement more traditional shocks as drivers of aggregate fluctuations. By design, those studies focus on firms’ reaction to exogenous forces that generate negative skewness or countercyclical volatility – they do not attempt to understand those forces or how policy might affect them.

This paper takes the alternative route: it highlights a nonlinear mechanism that *endogenously* shapes the distribution of employment growth and documents its relevance using Census micro data. The basic idea is that firms use concave hiring rules: when facing firm-level shocks to, say, productivity, they respond more to bad shocks than to good shocks. Concave hiring rules offer a unified explanation for why negative skewness and countercyclical volatility should be present both in the cross section and in the time series at different levels of aggregation. A firm’s environment – including policy – thus matters for higher moments of employment growth to the extent it changes the optimal hiring rule.

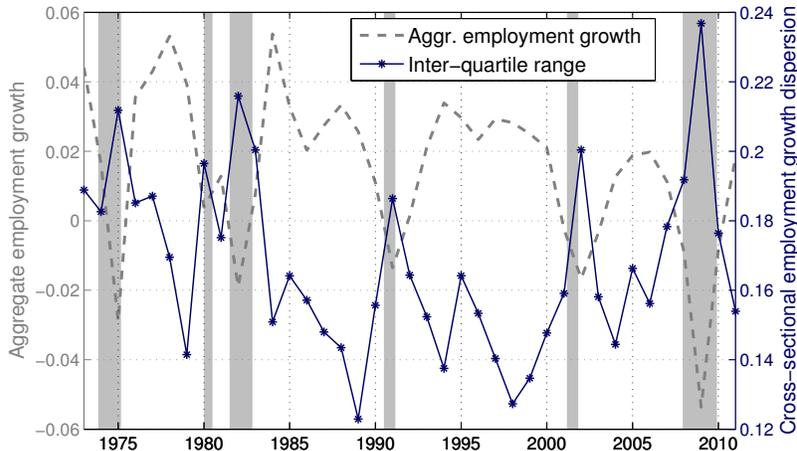
With concave hiring rules, negative skewness and countercyclical volatility at the firm level emerge in response to many shocks and need not reflect new exogenous forces. Indeed, a concave hiring rule says that a firm responds more when it receives a bad shock, compared both to other firms at the same date and to its own past behavior. We thus obtain negative skewness in both cross section and time series even if firm level shocks are not skewed in either dimension. Moreover, a bad aggregate shock lowers the average firm level shock and leads the typical firm to respond more. The cross sectional dispersion of employment growth thus increases in bad times even if the volatility of firm-level shocks is unchanged.

Concave hiring rules also explain why we observe negative skewness and countercyclical volatility at the aggregate level. Indeed, a bad aggregate shock makes the sum of all firms respond more sharply than it does after a good aggregate shock. It thus generates negative skewness of aggregate employment growth (“macro” skewness) and ensures that movements in aggregate employment

¹Similar patterns have been documented for a number of firm level variables; Bloom (2014) surveys the evidence.

growth are stronger in recessions (countercyclical “macro” volatility). Existing models that link countercyclical macro volatility to countercyclical cross-sectional (“micro”) volatility have relied on correlated shocks to the mean and volatility of productivity. With concave hiring rules, the link emerges endogenously and such shocks are not needed.

Figure 1: Time-varying employment volatility



Note: This figure plots aggregate employment growth in the manufacturing sector (left axis in gray) together with the year-by-year inter-quartile range (right axis in blue) of employment growth rates across firms in the sample of manufacturing establishments described in Section 3.

This paper starts from a simple model that illustrates the key mechanism and derives predictions for micro and macro moments. Our empirical analysis then proceeds in two steps. The first is to document volatility and skewness patterns for employment growth. It thus checks predictions of the model that do not depend on what shocks firms respond to. The second step studies firms’ responses to specific shocks, namely innovations to TFP.² We show that those responses are concave and transform symmetric shocks into skewed employment growth. Moreover, concavity is systematically related to volatility and skewness in the cross section of firms and is large enough to contribute significantly to aggregate volatility and skewness.

We use minimal structure to describe our mechanism – in particular, we do not take a stand on what makes hiring rules concave. One possibility is direct hiring costs: hiring new workers may entail costly search, whereas firing is free. Another is financial market imperfections: investment in new workers requires costs of external finance, whereas downsizing does not. In either case, regulation may matter for the strength of market frictions. Concavity can also obtain without adjustment costs: we show that decision makers who are averse to Knightian uncertainty (ambiguity) about

²Formally, we consider innovations to profitability, or revenue total factor productivity (TFPR as in Foster et al. (2008); Hsieh and Klenow (2009)). For simplicity we refer to this variable as TFP.

the quality of signals also find it optimal to respond more to bad news.³

While our main results are consistent with any candidate reason for concavity, our empirical findings of where concavity matters most provide some guidance for how to build nonlinear models that impose more structure. Most importantly, they suggest a focus on asymmetries at the establishment level. Indeed, we do not find strong evidence to connect concavity of the establishment level response to TFP shocks with the size of the parent company. At the same time, establishment level responses are more concave if the establishment is smaller, older, more capital intensive and if it employs more non-production workers. Finally, we find more concavity in industries that are more volatile and produce non-durable goods.

Confidential Census data on U.S. manufacturing establishments allow detailed micro level analysis of employment growth, controlling for time period, industry and other firm level properties. We confirm the well known facts that aggregate growth is negatively skewed and that both micro and macro volatilities are countercyclical. Our mechanism further predicts that employment growth should be negatively skewed in the cross section. In the data, the cross section of employment growth is indeed negatively skewed in all years.

We also provide several new results in the time series dimension. As our mechanism suggests, negative skewness is present not only for aggregates but also at the level of the industry and the individual firm. Moreover, rolling window estimates of volatility reveal countercyclical time series volatility at the industry and firm levels. While our main results are for manufacturing, we make use of the LBD database to show that the distribution of employment growth exhibits the same broad patterns for other sectors as well.

In principle, negative skewness of employment growth could obtain either because firms respond linearly (that is, symmetrically) to skewed (asymmetric) shocks, or because firms respond asymmetrically to (possibly even symmetric) shocks.⁴ For the manufacturing sector, we can obtain more direct evidence by considering firms' employment responses to TFP shocks. We construct establishment level TFP innovations and then run both nonparametric and nonlinear parametric regressions of employment growth on TFP innovations, controlling in various ways for other state variables that could be relevant to the firm.

We find that the relationship between employment growth and innovations to profitability is usually concave. On average, a firm faced with a typical negative (positive) TFP innovation – which corresponds to 18% lower (higher) output holding inputs fixed – decreases (increases) employment by 1.8% (1.2%). We find similar asymmetric hiring responses in the vast majority of parametric and non-parametric specifications that we consider, conditioning in particular on size, industry and time period. We conclude that at least one important source of shocks to firms, namely TFP

³Intuitively, ambiguity averse firms evaluate hiring decisions *as if* taking a worst case assessment of future profits. With ambiguity about signal quality, the worst case then depends on what the signal says: for a good signal, the worst case interpretation is that it is noisy, whereas for a bad signal the worst case is that it is very precise. Updating from ambiguous signals thus endogenously generates asymmetric actions.

⁴One piece of evidence in favor of asymmetric responses is the countercyclical movement of cross sectional dispersion – it would not follow from skewness of shocks alone. Indeed, if an aggregate shock shifts the mean of a skewed distribution of shocks and the employment response is linear, the volatility of responses need not change at all.

innovations, is propagated via asymmetric adjustment in firm behavior.

We provide two exercises to make a connection between the moments of employment growth we have documented and the estimated concave response to TFP shocks. In principle, those two sets of facts might be unrelated: since TFP shocks are not the only source of uncertainty facing firms, a concave response to TFP shocks may not be enough to shape moments of employment growth such as skewness. One way to assess whether there is a connection exploits the cross section of firms. If our estimated response reflects asymmetric adjustment to shocks more generally, then firms with more concave responses to TFP should also have more negatively skewed employment growth. We thus construct measures of concavity that capture how much of the variation in our non-parametrically estimated hiring rule of a firm is contributed by its non-linear terms.

We then compare properties of employment growth across groups of firms that differ in the concavity of their response to TFP shocks. We find that firms with more concave hiring responses indeed exhibit more negative skewness in employment growth relative to the moments measured for their TFP shocks. Moreover, those firms exhibit higher volatility in employment growth relative to TFP, consistent with the amplification predicted by our mechanism. Finally, we confirm that countercyclicality of dispersion is stronger for firms with more concave responses to TFP shocks. Taken together, these cross sectional results provide further evidence that our mechanism is important for hiring.

A second way to connect employment growth moments to our estimates of concavity is to ask what a concave response to TFP shocks contributes to overall negative skewness and countercyclical micro volatility. The starting point here is the distribution of TFP itself: consistent with the literature, we find that a one standard deviation negative shock to aggregate TFP goes along with a 7% increase in the micro volatility of TFP. Even linear responses by firms would thus generate some countercyclicality in employment growth dispersion. Our mechanism implies a much larger response: concavity alone generates an additional 20% spike in micro volatility. It further transforms and amplifies both average TFP shocks and volatility shocks to generate significant effects on higher moments of employment growth.

The estimated concave responses also contribute significantly to negative skewness in employment growth. In fact, the negative skew in employment growth stands in sharp contrast to the fact that TFP innovations have a nearly symmetric but slightly positively skewed density. It can therefore not be explained by a linear response to skewed TFP shocks. At the same time, the fitted value of employment growth explained by the concave response to TFP exhibits cross sectional negative skewness of -1.17 .

The paper is structured as follows. Section 2 illustrates the mechanism using a simple organizing framework. Section 3 introduces the data and describes the distribution of employment growth, with an emphasis on negative skewness. Section 4 turns to the cross sectional relationship between employment growth and TFP innovations and Section 5 illustrates the quantitative effects of that asymmetric relationship.

Related literature There is now a large literature on business cycle models with exogenous shocks to idiosyncratic or aggregate volatility. On the one hand, Bloom (2009), Bloom et al. (2012), Christiano et al. (2014), Arellano et al. (2010), Berger and Vavra (2014), Vavra (2014) and Schaal (2012) derive implications of cross sectional risk shocks for business cycles, borrowing and lending, consumption, price dispersion and the effectiveness of policy. On the other hand, Fernández-Villaverde et al. (2011), Gourio (2012) and Basu and Bundick (2011) study the effects of changes in conditional higher moments of aggregate TFP. Our paper does not formulate a full-fledged business cycle model; our goal is to zero in on firms’ asymmetric response as a mechanism to connect micro and macro volatility. We also emphasize that our mechanism is not incompatible with the presence of exogenous volatility shocks – instead, we think of it as an amplification mechanism that would be interesting to explore in the context of business cycle models.⁵

While some studies draw connections between micro and macro moments, they look at different stylized facts. For example, Carvalho and Gabaix (2013) show that the sectoral composition of the U.S. economy could have contributed to the Great Moderation, holding fixed the distribution of shocks within sectors. Our paper instead focuses on the business cycle frequency and heterogeneity within industries. Nimark (2014) derives an endogenous link between large (positive or negative) movements in aggregates and the dispersion of survey forecasts from a learning mechanism in which outliers are more salient. While learning could help explain the adjustment behavior we emphasize, it is critical for our mechanism that adjustment be larger for bad shocks, not simply for large shocks.

A recent literature has asked why dispersion in measured productivity is countercyclical. Kehrig (2015) documents countercyclical TFP dispersion and explains it with cyclical entry/exit and over-heads in production. Bachmann and Moscarini (2011), Kuhn (2014) show how firms’ pricing decisions can lead to countercyclical measured productivity dispersion. Decker et al. (forthcoming) relate countercyclical productivity dispersion to cyclical investment in intangible production knowledge. Straub and Ulbricht (2016) study theoretically how nonlinear transformations can induce changes in volatility. While our paper shares with these studies an emphasis on endogenous movements in micro volatility, our focus is on employment as well as on the connection between micro and macro moments as well as the role of skewness. Importantly, we also document the empirical relevance of a unifying endogenous mechanism using Census micro data.

Several recent papers have studied the macroeconomic effects of asymmetric firm decision rules, in particular the resulting asymmetry in the business cycle itself. For example, George and Kuhn (2014) consider costly capacity choice and investment irreversibilities, Senga (2016) analyzes a heterogeneous-firm model with learning and Ferraro (2013) studies a search and matching framework with heterogeneous workers. They show that asymmetric responses can help explain “depth asymmetry” (deep recessions vs. meek booms) and “steepness asymmetry” (rapid recessions vs. smooth booms) – concepts introduced Sichel (1993) and studied for aggregate employment by

⁵Ludvigson et al. (2016) estimate a structural vector autoregressive model and argue that most of conditional volatility of the unforecastable component in real activity is an endogenous response to exogenous disturbances, in particular, shocks to real activity or shocks to uncertainty about financial variables.

McKay and Reis (2008).⁶ Our mechanism is consistent with asymmetry of the business cycle and creates a connection between aggregate asymmetries and micro and macro behavior of volatility and skewness.

While our empirical work focuses on how concave decision rules amplify bad shocks, our propositions also imply the converse mechanism if policy rules are convex. This perspective helps connect some results in the literature on investment dynamics. Indeed, the cost of reducing capital at the firm level is likely higher than the cost of increasing it (see for example, Caballero et al. (1995) and Ramey and Shapiro (2001)). With a convex investment policy function, our analysis predicts positive skewness as well as procyclical dispersion. In fact, Bachmann and Bayer (2014) show that investment rates exhibit procyclical cross-sectional dispersion as well as positive time-series skewness. Moreover, Bachmann et al. (2013) document a procyclical aggregate volatility of investment.⁷

2 Macro & micro moments with asymmetric adjustment

To illustrate the basic mechanism, the following minimal framework is sufficient. Consider hiring decisions by a continuum of firms. An individual firm’s information about its future profitability is summarized by a sufficient statistics s . For simplicity, we refer to s as the firm’s “signal,” although in practice it might represent information gleaned from many separate sources.

We assume that firms’ signals can be decomposed into a common component a and an idiosyncratic component ε :

$$s = a + \varepsilon. \tag{1}$$

The idiosyncratic component is independent and identically distributed across firms with mean zero and distribution function G_ε . We can thus think of (1) as representing both the distribution of an individual firm’s signal and the cross sectional distribution of signals.

Firms respond to signals about future profitability by changing employment. We assume that all firms follow the same decision rule

$$n = f(s), \tag{2}$$

where n denotes employment growth rate and the function f is smooth, strictly increasing and strictly concave. The function f describes both how beliefs are formed given the realized signal s and the optimal employment choice given beliefs.

The assumption of concavity reflects asymmetric adjustment: firms respond less to good signals than to bad signals. We take asymmetric adjustment as given; it can be due, for example, to an asymmetric hiring cost function through which it is more costly to hire than to fire. It can

⁶Negative skewness in growth rates of macroeconomic aggregates such as consumption and output is also important in the recent literature on the effect of disasters for asset pricing, following Rietz (1988) and Barro (2006).

⁷In principle, investment might depend on firm level shocks other than TFP that are themselves skewed. At the same time, Bachmann et al. (2014) do not find skewness in innovations affecting investment. Their result also points towards endogenous propagation of shocks as a source of skewness.

also be due to belief formation: Appendix A provides an example of information processing under Knightian uncertainty to show that asymmetry can obtain without asymmetric adjustment costs.

Our setup does not take a stand on the relationship between the signal and actual profitability. In general, there may be some true profitability innovation π , say, which itself has aggregate and idiosyncratic components, and firms respond to private noisy signals about their own profitability. In this case, a would contain the aggregate component of π plus noise in common signals that is correlated across firms and ε would contain the idiosyncratic component of π plus noise in idiosyncratic signals that is uncorrelated about firms.

The relative share of noise and “fundamental” is not important for our argument. We only need that firms respond in a concave way to the random variables s – this feature alone implies the restriction on the cross section and time series of firm actions that we are interested in. In particular, the model is thus consistent with the common assumption that firm level profitability follows a Markov chain and firms forecast future profitability given their knowledge of current profitability, with or without additional noise in signals.

In general, firm decisions may depend on state variables other than the signal on the next innovation to profitability. For example, firms might respond differently to shocks depending on fixed characteristics like industry, or variable characteristics like size, the level of profitability or the deviation of employment from a target level. For now, these additional features of the firm decision at a point in time are all subsumed in the function f . In other words, the class of firms we study in this section is identical along all these dimensions.

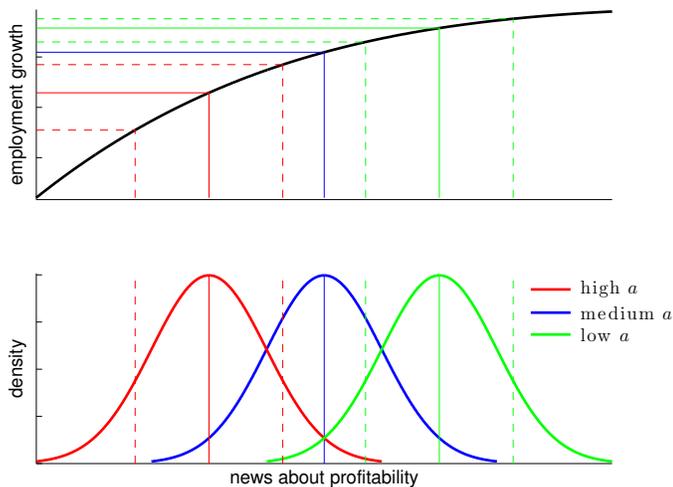
In order to describe common shocks to firms, we condition on the aggregate component a . Let $G_n(n|a)$ and $G_s(s|a)$ denote the conditional cdf’s of employment growth and signals, respectively, given a . We will refer to high values of a as representing “good times,” that is, times when firms on average receive good news about profitability. An implication of our assumptions is that the conditional variance $var(s|a)$ is independent of a , so good times are only reflected in a high mean signal. This is helpful to zero in on the endogenous link between macro and micro volatility that is driven by asymmetric adjustment. We discuss interaction of asymmetric adjustment and exogenous shifts in volatility further below.

2.1 Implications for micro and macro volatility of employment

The main intuition for countercyclical volatilities can be seen in Figure 2. The top panel plots the concave response function f , with the signal realization on the horizontal axis and employment growth on the vertical axis. The bottom panel shows three densities of signal realizations $g_s(s|a)$, distinguished by a shift in the mean signal. Taking the middle (blue) density as a reference point, a shift to the left (red) density is an arrival of bad times, whereas a shift to the right (green) density is an arrival of good times.

The figure shows how asymmetric adjustment helps make both macro and micro volatility countercyclical. Consider first macro volatility: the solid horizontal lines in the top panel represent mean employment growth for the three densities. They show how bad news generate larger aggregate

Figure 2: Employment growth and signals



Note: Figure plots the (homoskedastic) distribution of signals about future profitability (bottom panel), once centered around a negative aggregate mean signal (red), once around a neutral aggregate signal (blue) and once for a positive aggregate signal (green). The top panel displays how the concave hiring rule (black) transforms symmetric and homoskedastic signals into asymmetric and heteroskedastic employment responses.

responses: the change in aggregate growth in response to bad times (the difference between the horizontal red and blue solid lines) is larger than the change in growth in response to bad times (the difference between the horizontal green and blue solid lines).

To illustrate changes in micro volatility, the dotted lines in both panels show interdecile ranges for signals (along the horizontal axis) and employment growth (along the vertical axis in the top panel). The point here is that concavity of the response function accentuates dispersion in signals in bad times while it attenuates it in good times. The countercyclical cross-sectional volatility is easiest illustrated for interquantile ranges, but this property is inherited by other measures of dispersion such as the cross-sectional variance.

Beyond the specific example of Figure 2, the properties of countercyclical macro and micro volatility require only the signal structure and concavity of the response function. We summarize this result in the following proposition, with proofs detailed in Appendix B:

Proposition 1 (Macro & micro volatilities) For any two aggregate shock realizations $a < a'$,

1. the sensitivity of the aggregate action with respect to the aggregate shock is higher at a :

$$\left. \frac{d}{d\tilde{a}} E[n|\tilde{a}] \right|_{\tilde{a}=a} > \left. \frac{d}{d\tilde{a}} E[n|\tilde{a}] \right|_{\tilde{a}=a'},$$

2. the cross sectional variance is higher at a :

$$\text{var}(n|a) > \text{var}(n|a'),$$

3. the interquantile range for any two quantiles \underline{x} and \bar{x} is higher at a :

$$G_n^{-1}(\bar{x}|a) - G_n^{-1}(\underline{x}|a) > G_n^{-1}(\bar{x}|a') - G_{\Delta e}^{-1}(\underline{x}|a').$$

Figure 3 shows the connection between micro and macro volatilities in simulated time series data. The red dotted line is a particular sequence of aggregate news a_t , drawn from a symmetric, homoskedastic distribution. For each realization of a_t , we compute the model implied aggregate employment growth $E[f(a_t + \varepsilon) | a_t]$ – shown as the solid blue line – and the interquartile range of the cross sectional distribution $G_n^{-1}(0.75|a) - G_n^{-1}(0.25|a)$, shown as the solid green line. The latter two (blue and green) lines move against each other, reflecting the countercyclicality of micro volatility: the interquartile is wide when aggregate employment growth is low.

Moreover, comparison of the red and blue lines shows that the latter has larger movements when employment growth is low, whereas the movements are quite similar when employment growth is high. Asymmetric adjustment thus translates homoskedastic shocks into heteroskedastic responses – if we measured the variance of aggregate employment growth over subsamples, we would obtain larger numbers in low growth periods. At the same time, the figure shows how asymmetric adjustment translates symmetric shocks into negatively skewed responses. This is another general property of our setup to which we turn next.

2.2 Implications for micro and macro skewness of employment

Figure 2 above suggests that asymmetric adjustment also induces skewness in the cross section and the time series. In particular, the distribution of employment growth responses should be more negatively skewed than the underlying signal distribution. We define skewness using two standard measures. One is the Fisher-Pearson coefficient of skewness, a moment based measure defined as a ratio of third and second moments. For a random variable x , the coefficient of skewness is

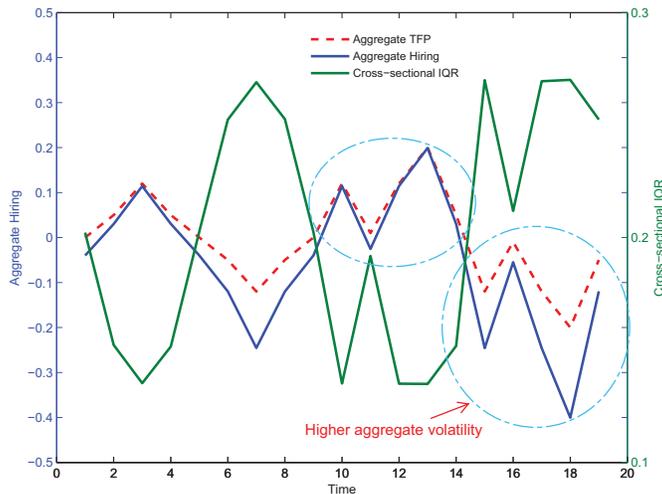
$$\gamma(x) = \frac{E[(x - E[x])^3]}{\text{var}(x)^{\frac{3}{2}}}. \quad (3)$$

The second measure is the Kelley skewness⁸, based on the distribution's percentiles and as such more robust to outliers, which is defined as

$$k(x) = \frac{x^{p90} + x^{p10} - 2x^{p50}}{x^{p90} - x^{p10}}. \quad (4)$$

⁸Kelley (1947), p. 250, modifies the skewness measure proposed by Bowley (1901) to use the 90th and 10th percentile instead of the top/bottom quartile.

Figure 3: Time-varying volatility and negative skewness in simulated data



Note: This figure displays aggregate employment growth and the cross-sectional inter-quartile of employment growth in simulated data. Downturns in aggregate employment are deep while upturns are mild thus creating negative time-series skewness. In recessions the cross-sectional dispersion widens and, as highlighted by the blue circles, aggregate volatility is higher than in booms.

where x^{pN} denotes the N th percentile of the distribution for the random variable x . The following proposition states formally that a concave response function induces skewness, measured either as the coefficient of skewness $\gamma(x)$, or as the Kelley skewness $k(x)$.

Proposition 2 (Micro and macro skewness)

1. *Micro skewness:*

- (a) *For any aggregate shock a the coefficient of skewness of the cross sectional distribution of employment growth, $\gamma(n|a)$, is lower than the coefficient of skewness of the cross sectional distribution of signals, $\gamma(s|a)$.*
- (b) *The same relationship holds for the Kelley skewness: $k(n|a) < k(s|a)$.*

2. *Macro skewness:*

- (a) *The coefficient of skewness of aggregate employment growth, $\gamma(E[n|a])$, is lower than the coefficient of skewness of the aggregate signal $\gamma(a)$.*
- (b) *The same relationship holds for the Kelley skewness: $k(E[n|a]) < k(a)$.*

Proposition 2 makes a statement about skewness *in general*, but it is silent on the cyclical movements in skewness. It is then natural to ask whether asymmetric adjustment implies systematic

movements in skewness together with movements in micro volatility. To see that movements in skewness are not implied by concavity of f alone, consider the example of a negative exponential response function $f(s) = -e^{-s}$, together with normally distributed noise. The conditional mean of employment given a is then $e^{-a+\frac{1}{2}\text{var}(\varepsilon)}$ and the k th centered moment conditional on a is given by

$$E \left[(f(s) - E[f(s)|a])^k \middle| a \right] = e^{-ka} E \left[\left(-e^{-\varepsilon} + e^{\frac{1}{2}\text{var}(\varepsilon)} \right)^k \right].$$

It follows in particular that the coefficient of (3) is independent of aggregate news a .

The special feature of the negative exponential response function is that curvature as measured by the coefficient of absolute risk aversion $-f''(s)/f'(s)$ is the same for all s . The next proposition provides a more general connection between curvature and skewness.

Proposition 3 (Cyclicalilty of skewness)

1. For any two aggregate shocks $a < a'$, the coefficient of skewness $\gamma(n|a')$ of the cross sectional distribution of employment growth at a' is higher than (lower than, equal to) the coefficient of skewness $\gamma(n|a)$ of cross sectional distribution of employment growth at a if the coefficient of absolute risk aversion $-f''(s)/f'(s)$ is decreasing in s (increasing in s , constant).
2. The same condition on the coefficient of absolute risk aversion applies for the cyclicalilty of the Kelley skewness of the cross sectional distribution of employment growth.

Intuitively, changes in skewness derive from changes in curvature in the relevant range of signals, in contrast to changes in volatility that derive simply from changes in the slope of the response function. It is therefore possible to have countercyclical volatility combined with countercyclical skewness, that is, in bad times the distribution of employment growth can be more dispersed but less negatively skewed. This is true for example if the response function is quadratic and thus exhibits increasing absolute risk aversion. We conclude from these results that our basic mechanism does not imply definite predictions about changes in cross sectional skewness over time.

2.3 Employment growth and innovations to profitability

The basic mechanism of asymmetric adjustment works whenever there is a distribution of firm-specific signals that shifts in mean over the business cycle. Unfortunately, firms' signals are not directly observable. However, with some extra structure on what shocks firms respond to, we can derive a relationship between those shocks and the distribution of employment growth.

We follow a large literature in focusing on shocks to firm profitability. In particular, we assume that detrended profitability, denoted by Z_t^i , evolves according to an AR(1) process

$$Z_t^i = \rho Z_{t-1}^i + z_t^i = \rho Z_{t-1}^i + u_t^a + u_t^i \tag{5}$$

where the innovation z_t^i has an aggregate (u_t^a) and an idiosyncratic (u_t^i) part, both with mean zero.

When firms make hiring decisions relevant for date t production, they observe past profitability Z_{t-1}^i , as well as a signal s_t^i about the current innovation z_t^i . The simplest assumption about signals is that firms observe the innovations z_t^i perfectly, that is, $s_t^i = z_t^i$, but our mechanism works also if the signals are noisy.⁹ In addition to its signal, a firm's decision will typically take into account Z_{t-1}^i , as well as possibly the level of employment (that is, firm size), or other features of the production function captured for example by the firm's industry. Those characteristics are captured by the function f .

Our empirical approach, detailed in Section 4, estimates the degree of concavity of the response of employment growth to the firm's signals s_t^i . Since we do not observe firms' information sets, we allow for noisy signals with general representation

$$s_t^i = u_t^a + u_t^i + v_t^a + v_t^i, \quad (6)$$

where the idiosyncratic components are independent conditional on u_t^a . In terms of the notation used above in equation (1), the aggregate news a here corresponds to the common shock u_t^a plus the correlated noise v_t^a , while the idiosyncratic component ε subsumes both the idiosyncratic profitability innovation u_t^i and the idiosyncratic noise v_t^i .

Consider now the empirical exercise of running a nonparametric regression of employment growth on the innovation to the profitability. Suppose further that the econometrician controls for calendar time, the role of industry and firm specific variables, allowing for nonlinear impact of those variables. The econometrician will then recover the conditional expectation given the true innovation $u_t^a + u_t^i$:

$$g(u_t^a + u_t^i) = E[f(s_t)|u_t^a + u_t^i]. \quad (7)$$

The only random variable in the expectation is the idiosyncratic noise v_t^i . Both components of the true innovation are fixed since they are observed by the econometrician, whereas the correlated noise v_t^a is fixed since it is common to all firms.

What is the effect of unobserved noise v_t^i for inference about the underlying response function f ? Without any noise, the estimated g function recovers exactly the desired response function f . Suppose instead that there is idiosyncratic noise but that it is independent of the aggregate innovation u_t^a . The proof of Proposition 1, part 1 then implies that the conditional expectation function g is concave (convex, linear) if the response function f is concave (convex, linear). This means that if an econometrician recovers a concave regression line g , he can rule out that the actual response function is linear or convex. A concave regression line is evidence in favor of the asymmetric adjustment that underlies our mechanism.¹⁰

The properties of g are informative about firm adjustment even if the noise v_t^i is not orthogonal

⁹Indeed, for the signal representation in equation (1) the idiosyncratic component u_t^i serves the role of generating dispersion in signals and the aggregate component u_t^a shifts the mean of the signals over the business cycle.

¹⁰We emphasize that this conclusion does not depend on homoskedasticity of the innovations u_t^i . In particular, it is true even if the variance of u_t^i depends on u_t^a , for example because innovations are more dispersed in bad times.

to the aggregate innovation u_t^a . An example of such dependence is that firms might receive more precise signals in good times. A second order Taylor expansion of the regression line delivers

$$g(u_t^a + u_t^i) \approx f(u_t^a + u_t^i) + \frac{1}{2} f''(u_t^a + u_t^i) \text{var}(v_t^i | u_t^a + u_t^i).$$

The curvature properties of the response g now reflect not only those of the decision rule f but also the interaction of asymmetric adjustment (captured by the second derivative f'') and the variance.

Several implications for the shape of f follow. First, concavity of g cannot be due to movements in signal precision alone. Indeed, if f is linear, then g must also be (approximately) linear. Observing concave g is thus evidence of asymmetric adjustment. Second, suppose that the decision rule is quadratic and the variance of noise is decreasing and convex in u_t^a , as would be the case if higher u_t^a increases the number of iid signals observed by the firm. It then follows that a convex (concave) decision rule implies a convex (concave) average response. The converse holds if the curvature properties of the decision rule do not change over the domain. We conclude that measuring a concave average response is indicative of a nonlinear and concave decision rule under quite general conditions.

3 Employment growth in cross section and time series

The concave decision rule illustrated in the previous section implies countercyclical volatility in the cross-section and in the aggregate over time as well as negative skewness in the cross section and over time. In this section, we first introduce our micro data sources and then check volatility and skewness properties for the distribution of employment growth both in the cross section and the time series. While we focus on the raw employment data in this section, Section 4 relates employment dynamics to estimated profitability measures.

3.1 Data sources

We use employment data at several levels of aggregation. The U.S. Bureau of Labor Statistics (BLS) provides monthly data on employment in the private economy, the ten major sectors and three-digit NAICS industries. We also rely on annual data on employment and technology growth contained in the NBER-CES Manufacturing Industry Database (NBER-CES) as well as confidential data on manufacturing establishments collected by the U.S. Census Bureau. The latter consist of the Annual Survey of Manufactures (ASM), the Census of Manufactures (CMF) and the Longitudinal Business Database (LBD).

We use Census CMF and ASM data to construct a large dataset of plants in the U.S. manufacturing sector. This panel spans the years 1972-2011, which allows us to study business cycle properties over six recessions including the “Great Recession” 2008/09. Every year, we observe about 55k establishments which total up to 2.1 million observations. We focus on the establishment as the unit of analysis and use the term “establishment” and “firm” interchangeably, following

convention in the literature.

We combine the ASM and the ASM portion of the CMF data (identified by establishment type $ET=0$), so to obtain a consistent longitudinal panel. By focusing on the ASM portion in all years, we automatically eliminate all administrative observations (identified by $AR=1$) which are imputed from industry means and might thus corrupt moments of the distribution we are interested in.

We further combine Census data with annual industry-level data from several publicly available sources: price deflators from the NBER-CES, various asset data from the the Capital Tables published by the BLS and the Fixed Asset Tables published by the Bureau of Economic Analysis (BEA). Most of this information is only needed to estimate productivity; for this purpose we follow [Kehrig \(2015\)](#), which contains details about the primary data and transformations required to estimate productivity.

We use the standard measure of employment growth: $n_t^i \equiv \Delta \log(L_t^i)$. This choice is helpful empirically as it is free of any specific metric. It is, however, not well-defined for firms that just entered or that are exiting the sample. Entry/exit can be due to economic birth and death of firms or due to the rotation of the ASM sample in years ending with 4 and 9. Both features are cyclical and thus not only affect the distribution in general, but have the potential to affect the cyclicity of employment dispersion we are interested in.

To check for robustness of our results with respect to cyclical entry and exit, we additionally construct employment growth rates as defined by [Davis and Haltiwanger \(1990\)](#)¹¹ and confirm our results using their growth rate. Note that this method is unable to distinguish between units that exit the economy or units that continue to exit but drop out of the ASM sample. To avoid outliers driving our results about dispersion, skewness and the employment-productivity link, we truncate the 1% tails of the employment and productivity (whose estimation is detailed in Section 4.1) data in the overall panel.

3.2 Cross-sectional employment growth dynamics

We focus on the changes in a firm’s total employment, defined as the sum of production and non-production workers. We focus on hiring rather than hours worked because the data report employment for all type of workers, while hours worked are only reported for production workers. This is not only a fraction of employment but it also reflects how the firm chooses overtime hours relative to normal hours. Furthermore, a firm’s employment variables in the Census data are considered of very high quality and there are virtually no missing values.

The asymmetric decision characterizing our model setup predicts that firms with negative signals reduce employment strongly, but firms with positive signals increase employment only slightly. As a consequence, this decision rule makes predictions about moments of the employment growth distribution across firms as summarized by Propositions 1-3: First, employment growth rates should be more spread-out when most firms receive negative signals, i.e. in recessions. Second, employment

¹¹This alternative measure of employment growth à la Davis/Haltiwanger is defined as $n_t^{i\text{ DH}} = 2(L_t^i - L_{t-1}^i)/(L_t^i + L_{t-1}^i)$ though [Bloom et al. \(2012\)](#) propose a variant where L_t^i is replaced by L_{t+1}^i .

growth should be negatively skewed on average. Third, this skewness is not necessarily cyclical. To check for these data features, we compute the second and third moments of employment across all firms in a given year and study the properties of the resulting annual time series which are summarized in Table 1.¹²

Table 1: Cross-sectional moments of employment growth

Moment	<i>Moment</i>			
	$StD_t(n_t^i)$	$IQR_t(n_t^i)$	$\gamma(n_t^i)$	$k(n_t^i)$
Long-run average	0.252	0.167	-0.477	-0.046
NBER booms	0.246	0.158	-0.495	-0.016
NBER recessions	0.272	0.200	-0.417	-0.146
Great Recession 2008/09	0.291	0.214	-0.301	-0.191
$Corr(dE_t^{aggr}, Moment_t)$	-0.689*** (0.076)	-0.637*** (0.120)	-0.253** (0.126)	0.698*** (0.083)
$Corr(\#boom\ qtrs/year_t, Moment_t)$	-0.317** (0.128)	-0.489*** (0.109)	-0.148 (0.113)	0.466*** (0.108)
Avge. no. of observations/year	40,400			

Note: Data are averages of those moments of the cross-sectional employment distribution plotted in Figures 4 and 5. dE_t denotes the growth rate of aggregate employment in the private economy, #boom qtrs/year denotes the number of expansionary quarters in the entire U.S. economy as defined by the NBER. Standard errors for the correlation coefficients are computed using a GMM procedure that corrects for heteroskedasticity and autocorrelation as in Newey and West (1987) and is adapted from Hansen et al. (1988). Census disclosure rules require rounding the number of observations to the nearest hundred.

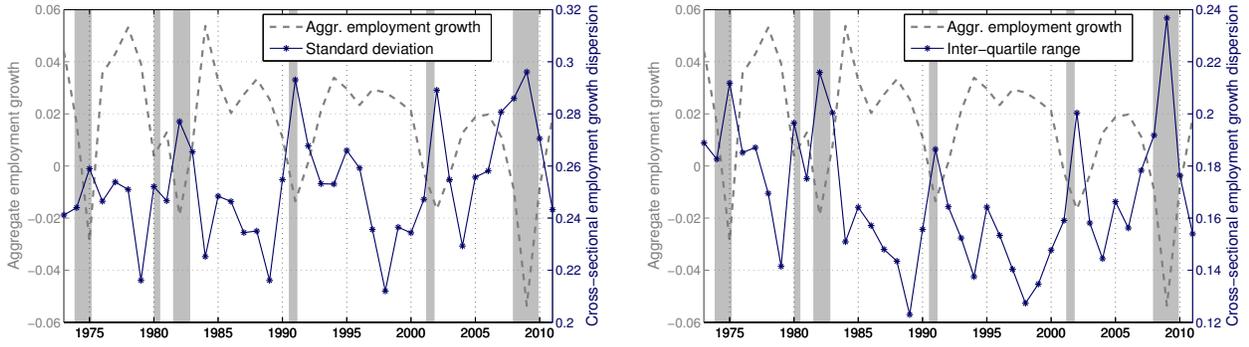
Cross-sectional employment growth dispersion

Figure 4 displays the year-by-year evolution of the standard deviation and the inter-quartile range (IQR) of the cross-sectional distribution of employment growth. Both of them show that firms differ a lot in their employment changes: on average, the firm at the top quartile grows employment by 16.7% more than the firm at the bottom quartile. The standard deviation across all firms is 25.2%. Both dispersion measures increase significantly in recessions. The inter-quartile range, for example, reaches 20.0% on average around a NBER recession compared to an average 15.8% in boom times.

In the Great Recession, the IQR experienced its sharpest increase ever, rising by more than half

¹²We share an interest in the role of nonlinear employment adjustment at the plant level with empirical work such as Caballero et al. (1997) who document a countercyclical cross-sectional dispersion of “desired employment.” To the extent that desired employment is some function of actual idiosyncratic signals about profitability, then the countercyclical dispersion that they document is consistent with a combination of a countercyclical dispersion in signals and, as our mechanism highlights, a concave desired employment function. While “employment gaps” are an interesting hypothetical object and relate to our work, we do not make the assumptions involved in their approach and instead analyze directly the measured employment response to identified profitability shocks, something that our more detailed data set allows us to do.

Figure 4: Dispersion of the cross-sectional distribution of employment growth



Note: This figure displays the unweighted year-by-year standard deviation and inter-quartile range of employment growth across firms for all $t=1973-2011$. That is, $StD_t(n_t^i) = \sqrt{1/(N_t - 1) \sum_{i \in t} (n_t^i - \bar{n}_t)^2}$ and $IQR_t(n_t^i) = n_t^{p75} - n_t^{p25}$ where \bar{n}_t denotes the mean employment growth, N_t the number of firms and n_t^{px} the x -percentile in year t . Following common procedure in the literature, we truncate the 1% tails of the overall panel to remove outliers.

its normal value before again returning to its long-run average in 2011. The standard deviation exhibits similar patterns. We also confirm that these patterns are present within 4-digit NAICS industries in order to avoid our results being driven by cyclical composition changes between industries with different but constant long-run dispersion. We also confirm that employment-weighted dispersion follows a similar pattern, so we know that firms in the tails of the distribution with very few employees are not driving our results.

Cross-sectional employment growth skewness

Our mechanism predicts that the employment distribution should be negatively skewed. Firms that shrink their employment should have an average rate of change that is larger in absolute value than that of firms which grow employment relative to the average employment growth. To check for this data feature, we compute two skewness measures across all firms in a given year and study the year-by-year properties of these time series.

Figure 5 displays the year-by-year evolution of the coefficient of skewness and Kelley Skewness:

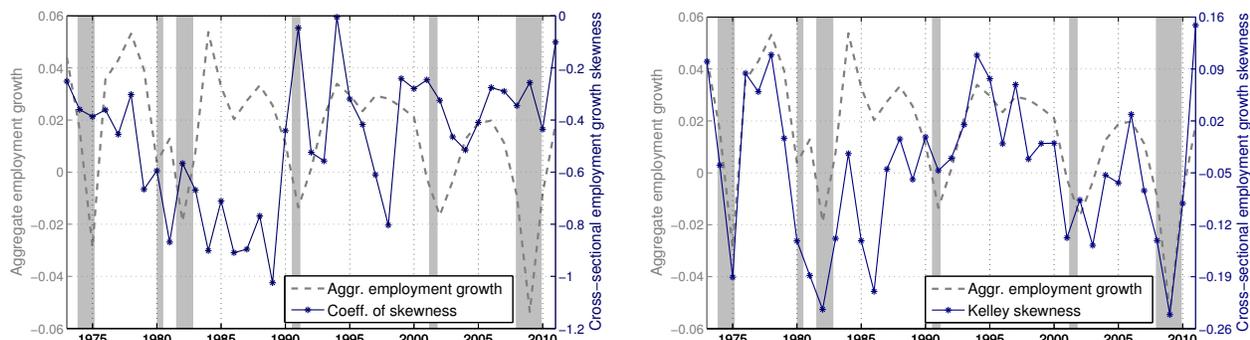
$$\gamma_t(n_t^i) = \frac{\frac{1}{N_t} \sum_{i=1}^{N_t} (n_t^i - \bar{n}_t)^3}{\left[\frac{1}{N_t} \sum_{i=1}^{N_t} (n_t^i - \bar{n}_t)^2 \right]^{3/2}}$$

$$k_t(n_t^i) = \frac{(n_t^{p90} - n_t^{p50}) - (n_t^{p50} - n_t^{p10})}{n_t^{p90} - n_t^{p10}}$$

where \bar{n}_t denotes the mean employment growth, N_t the number of firms and n_t^{px} the x -percentile in year t . Both the coefficient of skewness, $\gamma_t(n_t^i)$, and the Kelley Skewness, $k_t(n_t^i)$, are scaled so that they are dimensionless. The latter skewness measure is bounded between -1 and $+1$, which

correspond to the polar cases of a distribution which is degenerate between the bottom decile and the median ($k(\cdot) = +1$) and vice versa when the top decile coincides with the median ($k(\cdot) = -1$).

Figure 5: Skewness of the cross-sectional distribution of employment growth



Note: This figure plots the unweighted year-by-year employment growth skewness measures $\gamma(n_t^i)$ as well as $k(n_t^i)$ across all firms for all $t=1973-2011$. Further details as described in the notes to Figure 4.

As Table 1 shows, both measures are negative on average. The long-run average of the coefficient of skewness is -0.477 , that of the Kelley skewness is -0.053 . The latter means that the left tail of the distribution (between bottom decile and median) is $11\% = (1+0.053)/(1-0.053)$ more spread-out than the right tail (between top decile and median).¹³ The negative cross-sectional skewness is also consistent with an asymmetric hiring rule. As we did above with the dispersion measures, we confirm that no outlier industry or firms at the tails drive the negative employment skewness.

The coefficient of skewness does not display clear cyclical behavior. While Table 1 suggests that it is slightly countercyclical, the correlation coefficient with aggregate employment is barely significantly different from zero. In Appendix C.1.3 we present comprehensive evidence on the cyclicity of employment growth moments for several other measures of the business cycle. That analysis confirms the acyclicity of the coefficient of skewness. At the same time, the Kelley skewness does appear significantly procyclical. In NBER recessions it decreases to -0.151 which means that the distribution between bottom decile and median is 36% more spread out than the distribution between top decile and median. The plausible presence of other unobserved shocks that influence hiring, even if also associated with concave decision rules but of different shapes, may contribute to a complex cyclical pattern of the cross-sectional skewness.¹⁴

Panels A.1 and A.2 in Table 9 in the appendix further reports the same cross-sectional moments

¹³Davis and Haltiwanger (1990) report a different but related fact: about 35% of total job creation is caused by firms with strongly positive employment growth ($n_t^{i\text{ DH}} > 0.666$) while strongly negative employment growth ($n_t^{i\text{ DH}} < -0.666$) accounts for 45% of job destruction. This suggests either that “strongly contracting” firms shrink employment more than “strongly expanding” firms increase employment or that “strongly contracting” firms are larger than “strongly expanding” firms.

¹⁴The ambiguous cyclicity of the two skewness measures suggests that the condition for pro-/countercyclicity established in Proposition 3 is not satisfied. When we non-parametrically estimate the hiring rule in Section 4.3, we confirm that this is the case empirically.

of employment growth for each of the ten main sectors of the U.S. economy to illustrate that the patterns measured in the manufacturing sector is also present in the rest of the economy.

3.3 Time series employment growth dynamics

Our model predicts negative skewness and countercyclical volatility of employment growth not only in the cross section, but also in the time series, both at the level of the individual firm and at higher levels of aggregation. Indeed, Proposition 2 says that a concave hiring rule f transforms any sequence of firm level shocks s into a negatively skewed sequence of employment growth n . Moreover, Proposition 1 says that during bad times when the aggregate signal a is low, firms respond more to their signals and we should observe higher volatility at any level of aggregation.

We compute time series moments for individual manufacturing firms, 19 three-digit NAICS manufacturing industries, sectoral employment and the aggregate economy. We construct estimators of the time series standard deviation, denoted by Vol_t , and time series skewness, denoted by $Asym_t$, of employment growth within five-year rolling windows as follows:

$$Vol_t^i \equiv \sqrt{\frac{1}{4} \sum_{\tau=-2}^2 (n_{t+\tau}^i - \bar{n}_t^i)^2} \quad (8)$$

$$Asym_t^i \equiv \frac{\frac{1}{4} \sum_{\tau=-2}^2 (n_{t+\tau}^i - \bar{n}_t^i)^3}{(Vol_t^i)^3} \quad (9)$$

where $\bar{n}_t^i \equiv \frac{1}{5} \sum_{\tau=-2}^2 n_{t+\tau}^i$ is the average employment growth of firm i in the five-year window around t .

We limit our attention to rolling windows of five-year length due to the quinquennial sample rotation in the ASM and only consider firms that we observe for the consecutive five years. Longer windows would filter out too many high-frequency changes and thus cover up cyclical patterns we try to examine; in particular, we would miss the importance of the “Great Recession” towards the end of our sample.

Since the actions’ sensitivity to the shock is higher conditional on a negative shock, our mechanism predicts that for the average firm Vol_t^i should be countercyclical, and $Asym_t^i$ should fluctuate around a negative number. Unlike for the cross-sectional moments, we construct these time series measures at several levels of aggregation. This allows us to study if and how much micro-level patterns of employment growth wash out with higher levels of aggregation.

We construct these two measures for every manufacturing firm in the ASM, for employment growth aggregated to the 19 main 3-digit NAICS manufacturing industries, for employment growth of the ten sectors in the U.S. economy and for total private aggregate employment growth. For the first two, we report averages across firms and industries, respectively. Of course, we have to be mindful of the fact that our times series using the micro-level Census data span only 41 years, those for 3-digit NAICS industries only 25 years. In addition to that, a large literature in macroeconomics has documented and studied the “Great Moderation,” that is the declining time-series volatility of

aggregate macroeconomic and micro-level variables (see among others Davis et al. (2006)). To account for that feature, we take out a linear trend in both our time series measures. Unlike volatility, the time series skewness measure does not have a clear and significant trend at any level of aggregation. We still account for a trend because we do not want any business cycle results to be contaminated by long-run trends.

Figures 6 and 7 plot the time series of Vol_t and $Asym_t$, respectively, of these rolling windows for two samples: the average across our firms in the ASM and the aggregate economy. Patterns at the industry and sectoral level look similar, so we merely report them alongside the other results in Table 2. As in Table 1, we report the long-run and cyclical properties of these rolling windows at all four levels of aggregation.¹⁵

Table 2: Time series moments of employment growth

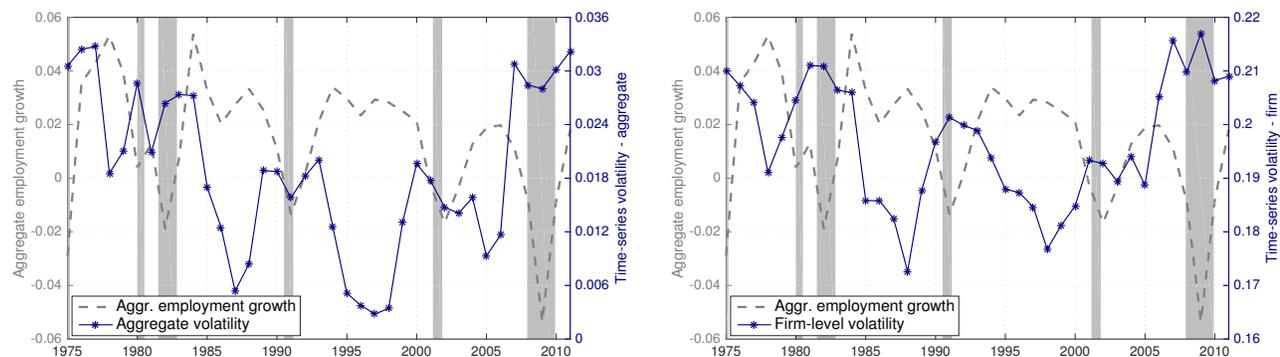
Moment	Aggregation level			
	Firm	Industry	Manufacturing	Aggregate
<i>A. Volatility</i>				
Long-run average	0.197	0.031	0.035	0.021
NBER booms	0.194	0.027	0.033	0.019
NBER recessions	0.207	0.047	0.043	0.025
Great Recession 2008/09	0.213	0.055	0.049	0.028
$Corr(dE_t, Vol_t)$	-0.468*** (0.124)	-0.526*** (0.121)	-0.148 (0.116)	-0.173 (0.109)
$Corr(\#boom\ qtrs/year_t, Vol_t)$	-0.435*** (0.107)	-0.326** (0.154)	-0.298*** (0.092)	-0.327*** (0.089)
<i>B. Asymmetry</i>				
Long-run average	-0.087	-0.411	-0.451	-0.459
NBER booms	-0.078	-0.333	-0.340	-0.429
NBER recessions	-0.113	-0.590	-0.713	-0.520
Great Recession 2008/09	-0.120	-0.840	-1.103	-0.835
$Corr(dE_t, Asym_t)$	0.333** (0.131)	0.0911 (0.1508)	0.114 (0.126)	0.068 (0.133)
$Corr(\#boom\ qtrs/year_t, Asym_t)$	0.361*** (0.117)	0.126 (0.190)	0.106 (0.249)	-0.010 (0.285)
Avge. no. of observations/year	4,500	19	1	1
Data Source	ASM	BLS	BLS	BLS
Time	1972-2013	1990-2015	1947-2015	1947-2015

Note: Firm-level Vol_t and $Asym_t$ measures are averaged across all firms in the ASM, the same procedure is applied for the industry-level analysis. Data at the industry level and above come from the BLS and are at annual frequency.

¹⁵Table 9 in the appendix reports the same moments at both the micro (Panels B.1 and B.2) and aggregate (Panels C.1 and C.2) level for each of the ten main sectors of the U.S. economy to illustrate that the patterns measured in the manufacturing sector is also present in the rest of the economy.

Time series employment growth volatility

Figure 6: Time-varying volatility of employment growth at the macro and micro level



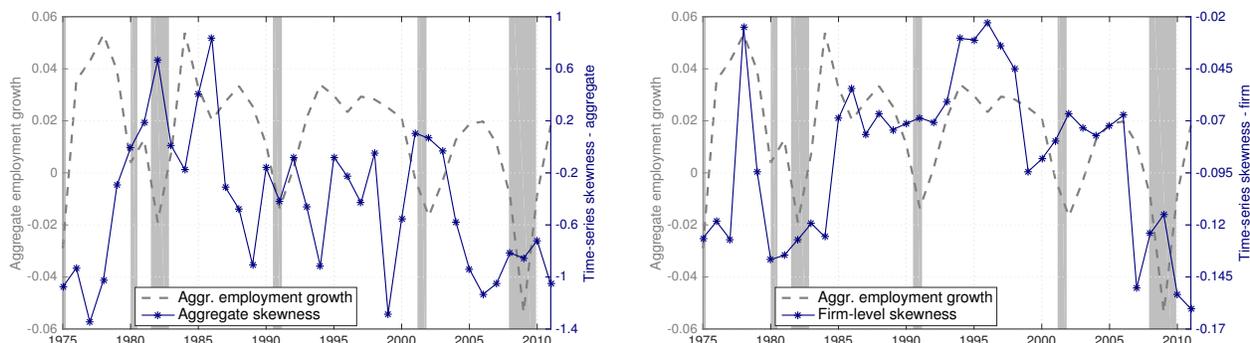
Note: Left panel: Vol_t measure for aggregate employment in the private economy with a linear trend removed; based on BLS employment data 1972-2013. Right panel: The year-by-year average across all firm’s Vol_t^i measure; based on ASM firm-level employment data 1972-2009.

As expected from the concavity mechanism, the time series volatility of employment growth, Vol_t , varies over the business cycle and is higher during and around NBER recessions at all levels of aggregation. For the time series at industry level and higher, our data allow us to examine if these patterns were even stronger in the “Great Recession” 2008-2010. We find that the Great Recession years were even more volatile than the typical NBER recession. Unsurprisingly, volatility is stronger on average at the firm level than the more aggregate levels. The difference across NBER expansions and recessions is not as strong as for firms as it is at the more aggregate levels – a feature that we attribute to noise at the level of many very small firms. Despite that difference in magnitude, time series volatility of both firm-level and more aggregate employment growth is strongly negatively correlated with the business cycle. We present aggregate employment growth and the number of NBER boom quarters per year as two representative measures of the business cycle in Table 2, but other measures such as GDP growth (displayed in Figure 15 in Appendix C.1.3) give similar results.

We further explore the countercyclical response of the aggregate economy by analyzing the conditional heteroskedasticity in aggregate employment. To that end, we follow the econometric procedure developed in Bachmann et al. (2013) and compute the cyclicity of the squared residual of aggregate employment growth. We provide these details and results in Appendix C.1.1. There we show that these squared residuals obtained from an autoregressive econometric model tend to be negatively related with aggregate employment growth. As Figure 10 in Appendix C.1.1 shows, volatility is about twice as high for aggregate employment growth rates of -1.8% than for +1.8%. Consistent with the message based on time-series volatility within windows, this provides further evidence on the countercyclical macroeconomic volatility implication of our mechanism.

Time series employment growth skewness

Figure 7: Time-varying skewness of employment growth at the macro and micro level



Note: Left panel: $Asym_t$ measure for aggregate employment growth. Right panel: averaged $Asym_t$ measure for firm-level employment growth.

Like in the cross sections, employment growth is skewed negatively over time. This means that employment contractions are sharper on average than employment expansions. This is true for both aggregate, sectoral, industry-level and firm-level employment growth. For the aggregate employment growth time series skewness is at -0.459 while this number is smaller for industry-level and firm-level analysis. For the typical firm the rolling-window skewness is only -0.1 on average. Given our construction, firms may be included in this average which are sampled merely for five years and then exit or are not sampled any more. If the ASM sample overrepresents young firms relative to old ones and the young firms tend to grow smoothly while older ones shrink in a volatile fashion, then this could bias our firm skewness measure towards negative infinity and vice versa for the opposite patterns. As a robustness check we limit our attention to a strongly balanced panel that contains those 1,900 firms which both exist and are sampled continually from 1972 until 2009.¹⁶ Focusing on this balanced panel, the negative skewness is more pronounced at -0.39 . This suggests that firms at the beginning and end of their life exhibit more symmetric growth patterns.

Unlike the cross-sectional evidence, the negative time series skewness of employment growth is stronger in recessions than in booms at any level of aggregation. This becomes even more evident during the Great Recession when the time series skewness of employment growth tends to be about twice as negative as it is in booms: the skewness of aggregate employment growth for example declines to -0.835 . Time series skewness of the average firm remains distinctly countercyclical when choosing employment growth or other variables as the measure of the cycle. This is not the case for the time series skewness at the industry, sectoral or aggregate levels where the correlation coefficient is insignificantly different from zero.

¹⁶We limit the sample to 2009 here since we want to compare the moments to those of TFP innovations which we can only construct until 2009 due to data limitations; none of the empirical results presented previously are significantly different.

4 The joint distribution of employment and TFP

In this section, we show that the firm level employment growth response to TFP shocks is concave. Section 4.1 describes the core empirical exercise: we construct innovations to the Solow residual, a measure of TFP and run regressions of employment growth on those innovations, using both nonparametric and parametric specifications that control for a battery of other variables.

We then present results in four steps. Subsection 4.2 reports cross section and time series properties of our estimated TFP innovations and compares them to those of employment growth. Subsection 4.3 presents the estimated concave response to TFP shocks. Subsection 4.4 looks at how concavity varies across a number of “data cuts.” Finally, Subsection 4.5 compares concavity and moments of employment growth across data cuts.

4.1 Constructing TFP shocks

All steps described in this subsection are done at the industry level to take into account that the dynamics of productivity may differ across industries. Unless otherwise noted, we consider industries at the 4-digit NAICS level. This choice allows for rich heterogeneity yet leaves enough observations in each industry to deliver precise estimates.

Solow residuals

We begin by deriving the Solow residual for every firm i and year t from the following standard Cobb-Douglas production function (in logs):

$$y_t^i = sr_t^i + \beta^k k_t^i + \beta^l l_t^i + \beta^m m_t^i + \beta^e e_t^i$$

where y_t^i is production (sales corrected for inventory changes and resales), k_t^i , l_t^i , m_t^i , e_t^i are real inputs of capital (structures and equipment), hours worked (production and non-production hours), materials and energy, respectively, and sr_t^i is the Solow residual. The production elasticity of production input $X = k, l, m, e$, labeled β^X , is equated to the revenue share of factor X . This is the only step where we define an industry at the 6-digit NAICS level, so the β 's are specific to the 6-digit industry.

Several advantages make this approach very suitable in our context. First, it is fairly free of structural assumptions and thus very general: we only need to assume that firms maximize profits and take factor prices as given. Using more involved structural estimates such as [Olley and Pakes \(1996\)](#) would require us to make timing assumptions about the arrival of information and the choice of inputs. Some of these would conflict with our own setup where choices are based on current signals that herald future productivity.

Identification of shocks

We assume that Solow residuals contain an aggregate growth trend, a common and a firm-specific fixed effect and a stationary component:

$$sr_t^i = gt + \bar{A} + \alpha^i + Z_t^i \quad (10)$$

where g denotes the long-run (industry-specific) growth rate, \bar{A} initial technology level (which will show up as long-run productivity differences across industries), α^i a firm-specific fixed effect such that $\sum_i \alpha^i = 0$. The distribution of Z_t^i over time is assumed to be stationary and have mean zero. Ultimately, we will be interested in TFP shocks, that is, innovations to Z_t^i . Given the assumptions on Z_t^i , we can identify g and \bar{A} as follows

$$\begin{aligned} E[d\bar{sr}_t] &= E[g + d\bar{Z}_t] = g \\ E[\bar{sr}_t] &= gt + \bar{A} + E[\bar{Z}_t] \end{aligned}$$

where $\bar{sr}_t \equiv N_t^{-1} \sum_i sr_t^i$ is the average Solow residual in t , $d\bar{sr}_t$ its growth rate, $\bar{Z}_t \equiv N_t^{-1} \sum_i Z_t^i$ is the common cyclical component of technology and N_t and is the number of firms in year t .

To identify TFP shocks, we assume the same AR(1) stochastic process for Z_t^i that we assumed in equation (5) in the model section: $Z_t^i = \rho Z_{t-1}^i + z_t^i$.¹⁷ After detrending the Solow residual in equation (10) and imposing the assumption of an AR(1) of Z_t^i , we get:

$$\begin{aligned} X_t^i &\equiv sr_t^i - gt - \bar{A} = \alpha^i + Z_t^i = \alpha^i + \rho Z_{t-1}^i + z_t^i \\ X_t^i &= \alpha^i(1 - \rho) + \rho X_{t-1}^i + z_t^i \\ &= \alpha^i(1 - \rho) + \rho X_{t-1}^i + u_t^a + u_t^i. \end{aligned} \quad (11)$$

Then we can estimate the objects α^i , ρ and z_t^i in equation (11) with a panel fixed effects regression.¹⁸

4.2 Asymmetric shocks or asymmetric responses?

Our model says that concave responses to TFP shocks induce negative skewness. If TFP shocks are the only source of variation, then Proposition 2 says that employment growth should exhibit

¹⁷We have also examined other shock processes. First, we decomposed Z_t^i into an industry-wide and idiosyncratic component (denoted by Z_t^{ind} and ζ_t^i , respectively) allowing for different persistences in either component $Z_t^i = Z_t^{ind} + \zeta_t^i = \rho^{ind} Z_{t-1}^{ind} + \rho^i \zeta_{t-1}^i + u_t^a + u_t^i$. While our estimates for ρ^i tended to be lower than those for ρ^{ind} , we cannot reject the null that the common and idiosyncratic persistence is the same for the majority of four-digit NAICS industries – a feature that we attribute to estimating these on only 37 periods.

We also examined whether or not the stochastic productivity component follows a random walk with drift: $sr_t^i = \alpha_i + Z_t^i = \alpha_i + g + Z_{t-1}^i + z_t^i$ but the estimated residuals are clearly autocorrelated which suggests that a random-walk model is misspecified.

¹⁸The fixed effect causes a well-known bias in the estimates of (11). Monte Carlo studies suggest the bias with 37 periods should be rather small. To check for robustness of our results, we also estimate (11) as suggested by [Arellano and Bond \(1991\)](#) and [Blundell and Bond \(2000\)](#); the resulting measure of TFP innovations is not only extremely similar to the one we obtain, the resulting employment growth-TFP innovation relationship is equally asymmetric; these results are detailed in the Appendix C.2, see in particular Table 10.

more skewness and countercyclical volatility than TFP shocks themselves. More generally, even if other shocks are relevant, a comparison of the moments for TFP shocks and employment growth is informative about what can possibly account for observed skewness in employment growth.

Table 3 reports annual cross-sectional moments of our estimated TFP shocks as well as employment growth. The main finding is that TFP innovations are barely skewed across firms in a given year. In addition, the cross-sectional standard deviation of a TFP shock on average is 0.178 – by coincidence it is very close to the inter-quartile range. A firm that is hit by a 1 standard deviation TFP shock thus produces almost 20% more output than the mean firm.¹⁹ By construction, the mean TFP shock is zero. In contrast, employment growth in manufacturing is negative on average as the manufacturing sector shrank over time.

Table 3: Summary Statistics

Moment	TFP Innovation z_t^i	Employment Growth n_t^i
Mean	0	-0.012
Std. Dev.	0.178	0.252
IQR	0.179	0.167
IDR	0.413	0.475
Skewness	0.072	-0.477
Kelley Skewness	-0.013	-0.046
N	1,536k	

Note: Data moments are time series averages of cross sectional moments of employment n_t^i and TFP innovation z_t^i 1973-2009.

Table 4 reports the time series skewness of TFP shocks and employment growth at different levels of aggregation. For the micro level we report these numbers separately for an unbalanced (Panel A.) and a balanced (Panel B.) panel of firms to account for the possibility that time series skewness of young firms that exit quickly differs from that of older firms where we get a more precise estimate of time series moments. We focus on the skewness of the entire time series. In the unbalanced panel the average time series skewness of employment growth is -0.187 while it is more negatively skewed in the balanced panel: -0.386. Skewness in both samples is slightly more negative when we take an employment weighted rather than the raw average. Firm-level TFP shocks u_t^i , in contrast, are basically not skewed over time regardless of panel and weighting. This means that, at the firm level, asymmetric employment patterns are not inherited from the underlying structural TFP shocks.

¹⁹Given our estimated shock process in equation (11), the cross-sectional standard deviation of TFP *levels* (as opposed to the above-reported TFP *innovations*) is 0.289 in a given year. This compares nicely to Syverson (2004) who estimates the cross-sectional inter-quartile range in the average industry to be 0.290 (measure “Total factor prod. 2” in Table 1); note that the inter-quartile range and standard deviation in our sample have basically the same value. In a similar procedure, we compute the inter-decile range of TFP *levels* to be 0.675 which is very close to 0.651 estimated by Syverson (2004).

Table 4: Time-series skewness of TFP innovations and employment growth

TS Skewness of...	TFP innov.	Empl. growth	N	Time	Data source
<i>A. Full panel of firms</i>					
Raw average	+0.047	-0.187	149,800	1972-2009	ASM
Weighted average	+0.027	-0.379			
<i>B. Balanced panel of firms</i>					
Raw average	+0.037	-0.386	1,900	1972-2009	ASM
Weighted average	+0.041	-0.554			
<i>C. NAICS-4 manufacturing industries</i>					
Raw average	-0.292	-0.937	86	1972-2009	ASM (aggr)
Weighted average	-0.299	-1.017	86	1972-2009	ASM (aggr)
<i>D. Manufacturing Sector</i>					
Average	-0.589	-0.996	1	1959-2009	NBER-CES
<i>E. Aggregate Economy</i>					
Quarterly	+0.397	-0.693	1	47:Q2-15:Q2	BLS &
Annual	-0.132	-0.949	1	1948-2014	Fernald (2014)

Note: Panel A. displays the average time-series skewness of employment and TFP innovations of a firm as described in (8). The TFP innovation z_t^i is defined in (11).

In Panel C. we examine the time series skewness of the average four-digit NAICS industry. Our estimation of TFP shocks in Section 4.1 also produced a TFP shock common to all firms in an industry, labeled u_t^a . We compared the time series skewness of that industry TFP shock to that of industry employment growth. While industry TFP shocks tend to be slightly negatively skewed (-0.292), industry employment growth is even more negatively skewed at -0.937. Again, weighting industries by their employment strengthens these patterns.

Panel D. repeats the analysis for the manufacturing sector using aggregated industry-level data from the NBER-CES. Again, manufacturing employment growth is more negatively skewed over time than sector-wide TFP shocks.

Finally, Panel E. displays the same statistics for the aggregate economy. As in Section 3.3 we compute aggregate employment growth rates from data on aggregate private employment from BLS which we annualize. To obtain a measure of an economy-wide TFP shock, we fit the same stochastic process for log TFP as in Section 4.1 on productivity data estimated in Fernald (2014). At the aggregate level TFP shocks are actually positively skewed while employment is negatively skewed both at quarterly and annual frequency.

At any level of aggregation and any available frequency we find that employment growth is more negatively skewed than TFP innovations which are not even skewed at all at the firm level. This means that asymmetric hiring and firing behavior are not due to asymmetric TFP shocks. Instead, the evidence is in line with by and large symmetric TFP shocks and a firm policy which translates these symmetric shocks into asymmetric employment responses as predicted by Propositions 1-2 in

Section 2. In the following section, we proceed to non-parametrically estimate this hiring rule.

4.3 Employment and TFP: non-parametric evidence

We now examine the average response of employment growth to TFP innovations. We estimate equation (7) from Section 2.3:

$$n_t^i = g(z_t^i) = E [f(s_t^i)|z_t^i] \quad (7')$$

As econometricians, we do not observe the firm’s relevant signals. However, as detailed in Section 2, we can recover the conditional expectation $g(\cdot)$ of the hiring decision $f(\cdot)$. This average response is conditional on what the econometrician measures, namely the observed TFP innovations over which the firm has (possibly noisy) signals.²⁰ In Section 4.1 we have estimated the current TFP shock z_t^i , which is observed by the firm. Because TFP is autocorrelated, the firm understands that the current TFP shock also signals higher future TFP and acts by changing employment, n_t^i .

We want to estimate (7') with the least restrictions possible. We therefore start with a non-parametric approach. Its key advantage is flexibility in the shape of the relationship between employment growth and TFP – we only require that the response function is smooth. A difficulty with a nonparametric approach is that it is hard to control for other state variables that might matter for the strength of an asymmetry of $f(\cdot)$ or that may lead to a spuriously asymmetric estimate of $f(\cdot)$ merely driven by composition bias.

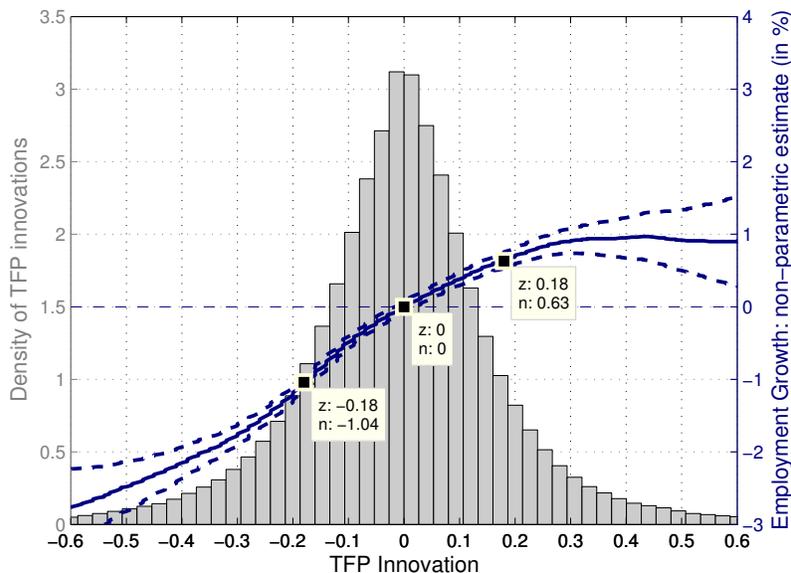
To address the potential bias due to the effect of other state variables, we partition the data into subsamples and estimate (7') for each subsample separately. We then aggregate the estimates for each subsample into one representative estimate. In this section, we divide firms into subsamples by size. Size is an important characteristic because the estimated asymmetry differs strongly along the size dimension compared. To account for differences in $g(\cdot)$ along other characteristics, we have repeated this exercise many times on different ways to partition the data and we report results from these different partitions in Section 4.4.

Figure 8 displays our non-parametric estimate of (7'). The estimated regression line corresponds to the function $g(\cdot)$, which reflects the recovered employment growth given technology innovation $z_t^i = u_t^a + u_t^i$. The solid dark blue line displays the mean employment change. The main takeaway is that employment growth responds more strongly to negative TFP innovations than to positive ones. Furthermore, the shape of the asymmetry is strictly concave over the domain of TFP innovations.

The concave pattern is strong in the relevant range of TFP shocks (illustrated by gray bars in Figure 8): after a typical positive one standard deviation TFP shock ($z_t^i = +0.178$) employment grows by 0.63% while it contracts by 1.04% for a negative one standard deviation TFP shock ($z_t^i = -0.178$). The employment response is also more sensitive for negative TFP innovations than positive ones: $g'(-z) > g'(z), \forall z \geq 0$. For example, the slope of the nonparametric estimate at a

²⁰One margin which a firm might use to faster adjust its employment to signals are temporary production workers. According to [Ono and Sullivan \(2013\)](#), however, the share of temporary workers makes up less than 5% of the production workers and they find evidence that temporary workers are usually hired to substitute regular production workers when the firm expects permanently lower output in the future.

Figure 8: Employment growth and TFP innovations



Note: Non-parametric regression of employment growth on TFP innovations for manufacturing firms. The non-parametric estimate is displayed as the solid dark blue line (right scale), dashed lines are 95% error bands. TFP innovations are obtained as described in Section 4.1, and their density is plotted in gray bars (left scale). Indicated data points display employment growth at a typical positive/negative (+1/-1 standard deviation) technology shock relative to no shock.

negative TFP shock is more than twice as strong compared to the slope at a positive TFP shock (0.073 compared to 0.029).

A non-parametric measure of asymmetry

Our final statistic captures the increase in volatility contributed by nonlinearity in the estimated employment response function g :

$$\phi_g \equiv 1 - \frac{g'(0)^2 \text{var}(z)}{\text{var}(g(z))} \quad (12)$$

If the estimated function g is close to linear and $g(z)$ is thus a scaled copy of TFP z , then ϕ_g is close to zero. In contrast, for strongly concave g , the variance of $g(z)$ is larger than that of $g'(0)z$, and ϕ_g is positive. For example, the concavity in overall hiring as displayed in Figure 8 implies that $\phi_g = 0.210$.²¹ The asymmetry is stronger when we estimate a firm's response for hours worked instead of employment. We emphasize that we measure hours worked only for production workers; their hours worked asymmetry measure is $\phi_g = 0.841$; only 15% of the variability in production

²¹By itself, the statistic ϕ_g measures the contribution of nonlinearity rather than concavity; for example $\phi_g > 0$ could be generated by convexity in g . Empirically, however, the estimated function $g(\cdot)$ is always concave.

hours worked is due to the linear portion in the hiring rule. This suggests that overtime hours may be very costly, so the firm is very reluctant to adjust this margin.

4.4 For which firms is concavity stronger?

In this subsection, we partition the data into subsamples that condition on characteristics of the firm, its parent firm²² or industry, one control at a time. This analysis is useful for two reasons. First, it shows that concavity of the hiring rule is a pervasive phenomenon that is evident in all subsamples.²³ Second, the various cuts of the data and the differential behavior of the asymmetry are informative about underlying structural reasons behind the concave hiring rule.

For each characteristic, detailed below, we split the data into octile bins and compute the asymmetry measure, given by ϕ_g in equation (12), over the firms inside that bin. Table 5 summarizes our findings. The fact that almost all cuts of the data still exhibit the asymmetric employment growth-TFP innovation relationship is reassuring and informative at the same time.

Our choice of controls is guided by three sets of candidate stories for why hiring rules might be concave. The first set of stories implies adjustment costs at the level of the parent company that controls the individual firms (or establishments) in our data. For example, models of financial frictions typically predict that increasing the number of workers is more difficult than firing because the parent firm faces a constraint on the amount of funding needed for an expansion. A typical implication is that larger parent firms face lower financing constraints. Indeed, firm size often relates to the size of a firm’s collateral and thus its debt capacity.

The first three columns of Table 5 consider three characteristics of the parent firm: employment, the number of establishments per firm, and whether the firm is publicly traded. Column (I) shows that, except for the first octile, parent firm size is not systematically related to asymmetry of the hiring function. This finding stands in contrast to a simple financial frictions story where the parent firm organizes external finance. Column (II) indicates that the number of units per parent firm is also not a predictive control. This result suggests that organizational frictions for hiring at the level of the parent firm are not a major issue. Finally, concavity is weaker for public than for privately held parent firms, as reported by column (III). Since a major characteristic of the former is an easier access to equity markets, the latter finding is consistent with some forms of financing constraints.

The second set of candidate stories calls for controls at the level of the individual establishment, shown in columns (IV)-(VII). The main idea here is that if hiring costs reflect costly labor search, they might be higher for workers that are more skilled or simply those that have to “fit in” to perform particular tasks. For a larger establishment, we might then expect that more workers are performing standardized tasks, so that hiring costs are smaller and the hiring rule is less concave.

²²“Parent firm,” identified by `FIRMID` in the data, is the organizational entity controlling what we call a “firm” (“establishment” in the data, identified by the `LBDNUM`).

²³The asymmetry is also robustly present across time periods. In particular, when we split the sample into periods of five years, we find that for each period the slope of the non-parametric estimate at minus one standard deviation technology shock is larger than the slope at a positive one standard deviation shock.

Table 5: Hiring asymmetry ϕ_g along various dimensions

Bin	Parent firm characteristics			Firm characteristics			Industry characteristics		
	(I) Size (empl.)	(II) No. units/ parent firm	(III) Public/ Private	(IV) Size (empl.)	(V) Share NP empl.	(VI) Age	(VII) Capital Intensity	(VIII) Industry volatility	(IX) Durables/ non-durables
1	0.735	0.288	0.210	0.829	0.483	0.206	0.000	0.256	0.339
2	0.370	0.307	0.399	0.676	0.195	0.259	0.202	0.279	0.631
3	0.202	0.240		0.360	0.208	0.211	0.146	0.208	
4	0.126	0.242		0.286	0.313	0.345	0.220	0.268	
5	0.230	0.128		0.217	0.304	0.434	0.304	0.354	
6	0.301	0.267		0.149	0.273	0.611	0.395	0.272	
7	0.235	0.215		0.163	0.459	0.509	0.449	0.583	
8	0.139	0.093		0.149	0.680	0.532	0.658	0.361	

Note: Bins are octiles in the distribution of the variable in question; Bin=1 corresponds to the lowest, bin=8 to the highest value. Exceptions from this rule are:

“Age:” bins correspond to 5-year windows (Bin 1=ages 1-5, bin 2=ages 6-10, etc.); the analysis in this column is limited to data from 1982 onwards when the measured average age in the ASM becomes stable.

“Private/Public:” Bin 1 = publicly traded firms, Bin 2 = privately held firms.

“Durables/Non-durables:” Bin 1 = firms in durable goods industries, Bin 2 = firms in non-durables goods industries.

Differences in hiring costs by skill motivate controlling for the share of non-production workers. It also motivates including capital intensity, assuming that capital is more complementary with worker skill. Finally, we consider establishment age since young firms may be on an expansion path: if more resources are always devoted to recruiting, then it is cheaper to respond to good shocks by hiring.

Column (IV) shows that the hiring adjustment is more linear for larger firms. The asymmetry in the first octile for size is similar across columns (I) and (IV), presumably driven by small firms with few employees. The comparison between the two size effects suggests that the relevant adjustment friction takes place at the firm/establishment level. Column (V) shows that firms with a higher share of non-production workers tend to feature more asymmetric hiring functions. Similarly, as presented in column (VI), there is substantially more concavity for firms with a higher capital intensity, a fact consistent with stronger frictions in the hiring technology for skilled labor that highly complements capital.

Finally, in column (VII), older firms are typically characterized by more concavity. This may be the result of a life-cycle phenomenon in which there is an initial period of aggressive expansion. In terms of accumulating experience on the labor market, this effect suggests that the hiring friction does not subside through a learning by doing mechanism in which the firm gets better in its labor search activities. In contrast, the age effect is consistent with a view in which younger firms employ recruiting technologies that reduce frictions adapting to current labor market conditions.

Our third set of data cuts considers industry characteristics. Column (VIII) shows that firms in more volatile industries tend to be associated with more concavity. A possible explanation is based on a model of information processing in which more volatility may lead to more ambiguity about the quality of signals received by the firm. As detailed in Appendix A, such a model predicts this relationship arising from an optimal hiring response that is stronger to bad news than to good news. Finally, column (IX) reports that there is less concavity in durable goods industries, consistent with inventories acting as a margin that smooths the hiring and firing needs at the firm level.

4.5 Concavity, skewness and volatility across data cuts

This subsection draws a connection between concavity of firms' response to TFP shocks and the properties of employment growth documented in Section 3. If TFP were the only source of variation, then concavity should be systematically related to, say, negative skewness in the cross section. The association should also be strong if either (i) TFP shocks are a major source of variation in hiring or (ii) concavity of the response to TFP shocks reflects frictions that also apply to the response to other shocks. Either way, a positive association would provide additional evidence that our mechanism is important.

We start from the data cuts in the previous subsection and check if the strength of the employment patters as predicted by Propositions 1-3 varies with the degree of concavity as estimated by ϕ_g . Our data cuts leave us with about 60 bins of data, and for each of these we know our concavity estimate as well as the features of the employment and productivity distributions. This allows us

to study the following five implications of concave hiring rules: The more concave,

1. the more dispersed employment growth,
2. the stronger employment growth dispersion varies over the business cycle,
3. the more negatively skewed employment growth.

When we study these predictions we are mindful of the possibility that distributional patterns in employment growth could be caused by the same patterns in the TFP shocks which differ across bins as well. We therefore always consider the relative moment, for example we consider the dispersion of employment growth in a given bin relative to that of TFP shocks in that same bin.²⁴ For the skewness measures in 3. we consider both the coefficient of skewness and Kelley skewness.

The scatter plots in Figure 9 confirm our predictions. Relative employment dispersion (plotted in the top left panel) is almost zero in symmetric bins while it rises to 12% on average in the most asymmetric bin. Similarly, relative employment dispersion varies more over the business cycle in asymmetric bins. Indeed, most of the boom-bust differences in the cross-sectional standard deviation of employment growth (plotted in the top right panel) reported in Table 1 seems to stem from bins that are asymmetric.

Bins where hiring is more asymmetric also exhibit more negatively skewed employment growth. This is true for both the coefficient of skewness (bottom left panel) and the Kelley skewness (bottom right panel). While bins with a symmetric hiring rule are almost not skewed, Kelley skewness becomes -0.15 on average in bins with the most asymmetric hiring rule. This means that the bottom decile is $(1 + 0.15)/(1 - 0.15) = 1.35$ as far away from the median as the top decile.

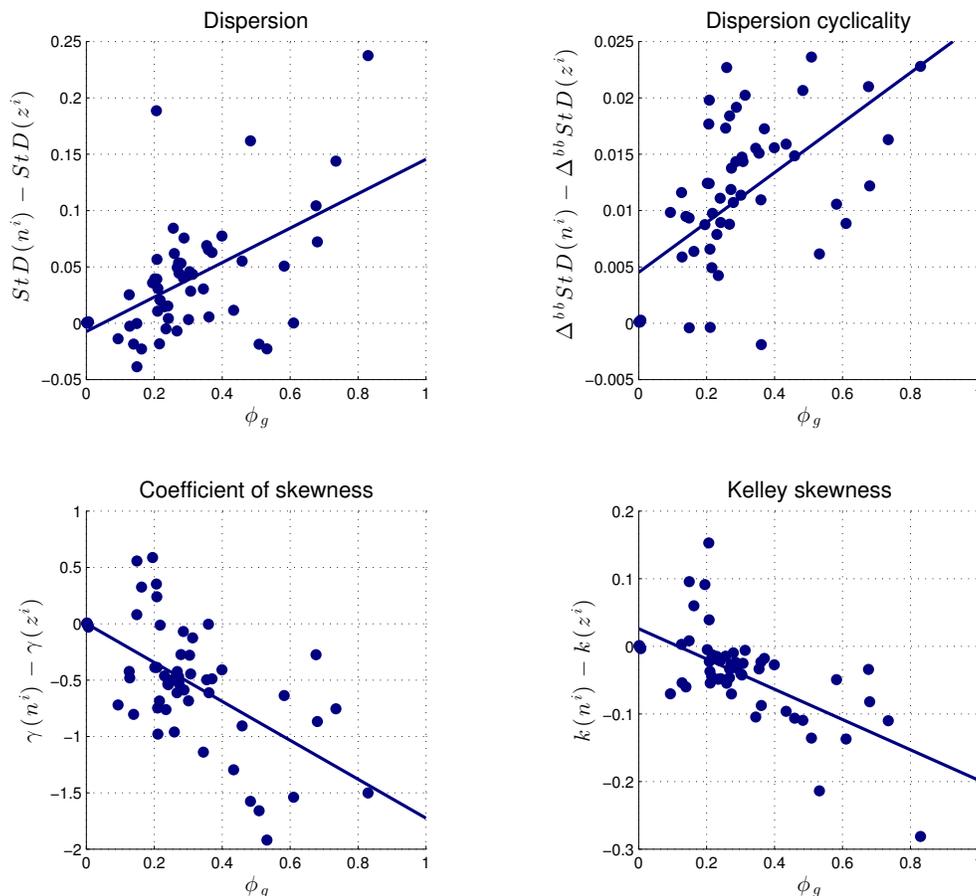
Our results are robust to cutting outliers and provide additional evidence of the impact of concave hiring rules on the employment growth distributions and responses. We omit the longitudinal implications of our propositions here because none of our data cuts would permit us to study that. We refer the reader to Tables 4 and 7 as evidence for longitudinal evidence for concave hiring rules.

5 Aggregate shocks and concave hiring rules

With concave hiring rules, shocks to average productivity could generate new facts on cross sectional and time series skewness and volatility that are inconsistent with traditional macroeconomic models. In this section, we investigate whether concavity in the response to TFP we have estimated at the firm level in Section 4 is sufficiently strong to contribute to those facts at level of an industry as well as the entire manufacturing sector. Moreover, we ask whether concave relationship between TFP and employment is also present at those higher levels of aggregation, and whether firm level responses can generate that relationship.

²⁴We experimented with both taking differences and ratios, but since TFP shocks are not skewed in many bins scaling employment skewness by (unskewed) TFP shocks would lead to extremely large relative values in many bins. Table 11 in Appendix C.3 displays volatility and skewness moments of TFP shocks along the same data cuts and bin definitions as in Table 5.

Figure 9: Hiring asymmetry and employment moments



Note: Data cuts from Table 5 where moments of the employment growth and TFP distribution have been estimated for each bin.

We start in Subsection 5.1 by approximating the concave shape of the empirical relationship between employment actions and TFP shocks plotted in Figure 8 by a parsimonious parametric form. The idea here is to help calibrate micro-founded macroeconomic models to our facts on non-linear hiring rules. Estimating hiring rules parametrically also helps to flexibly control for other state variables that could be relevant to the firm without falling prey to the curse of dimensionality or losing tractability. It thus serves as an additional robustness check on the results from the previous section.

In Subsection 5.2, we present results at the aggregate level. In Subsection 5.3, we then combine the estimated response functions with shocks to average productivity and compute the magnitude of the response. Finally in Subsection 5.4, we consider the effect of exogenous shocks to the volatility of TFP.

5.1 Employment and TFP: firm level parametric evidence

We focus on functional forms motivated by different models of asymmetric adjustment. Many models with adjustment costs deliver polynomial hiring rules. Other models, – in particular, the setup with ambiguity averse decision makers sketched in Appendix A gives rise to piece-wise linear hiring rules. We consider the general regression

$$n_t^i = \beta_0 + \beta^i + \beta_1 t + \beta_2 Z_{t-1}^i + \beta_3 l_t^i + \beta^\zeta \zeta_t^i \quad (13)$$

where t is a time trend, Z_{t-1}^i the lagged TFP level (reflecting our assumption of TFP following an auto-regressive process), l_t^i the logarithm of employment and c a constant. ζ_t^i is a function of the technology innovation aimed at capturing any possible asymmetries.

We examine the following specifications for asymmetries

$$\beta^\zeta \zeta_t^i = \begin{cases} \beta_4 z_t^i + \beta_5 (z_t^i)^2 & \text{Specification (I): concave,} \\ \beta_4 z_t^i + \beta_5 (z_t^i)^2 + \beta_6 (z_t^i)^3 & \text{Specification (II): non-monotonic asymmetry,} \\ \beta_4 z_t^i + \beta_7 z_t^i \mathbb{I}\{z_t^i < 0\} & \text{Specification (III): piece-wise linear.} \end{cases}$$

Specification (I) implies that employment growth is increasing and concave (increasing and convex) in TFP innovations if $\beta^4 > 0, \beta^5 < 0$ ($\beta^4 > 0, \beta^5 > 0$), specification (II) allows for more flexibility in fitting non-monotone relationships, specification (III) assumes a linear relationship, but has potentially different slopes for positive and negative innovations.

For each specification (I)-(III) we run a fixed effects panel regressions to account for persistent firm-specific factors in hiring. Since the non-parametric analysis above suggested that larger firms have a less asymmetric hiring policy, we also run a set of the same panel regressions (I)-(III) weighting each observation by its average employment. This should tell us if the quantitative relevance of asymmetric hiring goes away if we consider such a “relevance-based” hiring policy.

Table 6 displays the estimates of a FE panel regression. As expected, TFP innovations have a positive effect on hiring and negative innovations reduce hiring. Ignoring the non-linear terms, a typical positive (negative) TFP shock increases (reduces) employment growth by 1.5 percentage points. But we do find considerable evidence for hiring asymmetries: Across all specifications, employment contractions after negative TFP shocks are larger in absolute value than expansions after a similarly sized positive TFP shock. In the first three rows, all estimates of the non-linear terms are statistically significant. To assess if an actual employment expansion is significantly different from an employment contraction, we evaluate the typical employment response at $+1/-1$ standard deviation TFP innovation: $\beta^\zeta \times \pm StD(\zeta_t^i)$. The rows “Pos. Resp.” and “Neg. Resp.” at the bottom of Table 6 display this typical employment response. For example, the unweighted estimates for specification (I) imply an employment response after a positive TFP innovation of $0.0827 \times 0.18 - 0.0998 \times 0.18^2 = 1.16\%$.

We label responses as significantly asymmetric at the $X\%$ level if the point estimate for a positive

Table 6: Firm-level employment asymmetry

$$\begin{aligned} \text{Regressions (Ia/b):} \quad & n_t^i = \beta_0 + \beta^i + \beta_1 t + \beta_2 Z_{t-1}^i + \beta_3 l_t^i + \beta_4 z + \beta_5 z^2 \\ \text{Regressions (IIa/b):} \quad & n_t^i = \beta_0 + \beta^i + \beta_1 t + \beta_2 Z_{t-1}^i + \beta_3 l_t^i + \beta_4 z + \beta_5 z^2 + \beta_6 z^3 \\ \text{Regressions (IIIa/b):} \quad & n_t^i = \beta_0 + \beta^i + \beta_1 t + \beta_2 Z_{t-1}^i + \beta_3 l_t^i + \beta_4 z + \beta_7 z \mathbb{I}\{z < 0\} \end{aligned}$$

Estimate	(Ia)	(Ib)	(IIa)	(IIb)	(IIIa)	(IIIb)
	quadratic		cubic		piece-wise linear	
β_4	0.0827*** (0.0019)	0.0775*** (0.0053)	0.0959*** (0.0027)	0.1005*** (0.0093)	0.0347*** (0.0034)	0.0373*** (0.0147)
β_5	-0.0998*** (0.0062)	-0.0934*** (0.0190)	-0.0956*** (0.0063)	-0.0881*** (0.0198)		
β_6			-0.0901*** (0.0163)	-0.1685*** (0.0440)		
β_7					0.0947*** (0.0057)	0.0795*** (0.0217)
N	1,536k	1,501k	1,536k	1,501k	1,536k	1,501k
R^2	0.168	0.1258	0.168	0.1259	0.167	0.1257
Weighting	no	yes	no	yes	no	yes
Pos. Resp.	+1.16%	+1.06%	+1.36%	+1.38%	+0.62%	+0.64%
Neg. Resp.	-1.80%	-1.61%	-1.97%	-1.90%	-2.31%	-2.01%
Diff. sign.?	Yes***	Yes***	Yes***	Yes***	Yes***	Yes***

Note: *, **, *** on parameter estimates means significantly different from 0 at the 10%, 5%, 1% level, respectively. Yes***, **, * means that the point estimate of the negative response lies outside the 99%, 95%, 90% error bands, respectively, of the estimate of a typical positive response and vice versa.

response lies outside the X% confidence interval of the symmetric negative response and vice versa. Across all specifications we observe that the hiring asymmetry is significant at least at the 95% level and that the hiring asymmetry is quantitatively relevant: the typical negative response is at least 1.5 times as strong in absolute value as the typical positive response and varies from -1.8% to -2.3%.²⁵ The fit of all specifications seems fairly similar as suggested by a similar R^2 .

The employment-weighted regressions overall imply the same results. The hiring asymmetry is statistically significant and quantitatively almost as sizable as the one implied by the unweighted regression (it is smaller by about a tenth).

We conclude by discussing the other controls. We include the lagged TFP level because the same TFP shock has probably a different employment effect at a higher TFP level than at a lower one, especially since our regression setup considers the more persistent effects captured by changes in the employment growth rate. The lagged TFP level does enter positively which confirms our intuition. Employment matters positively for hiring which reflects the fact that large firms tend to hire more in general. As the negative coefficient on the time trend suggests, the already negative employment

²⁵Appendix C.4 further shows that an asymmetric hiring rule is not an artifact of composition effects caused by firm life cycle patterns or by labor-saving capital-embodied technological change.

growth rate in the manufacturing sector is accelerating over time – probably a consequence of increased outsourcing of manufacturing jobs abroad.

5.2 Employment and TFP: aggregate parametric evidence

In this section, we consider the contemporaneous relationship between TFP shocks and employment growth at higher levels of aggregation. The idea is to produce aggregate moments that help evaluate nonlinear business cycle models. Indeed, standard business cycle analysis judges models by how well they match unconditional second moments. To assess models that can generate our mechanism, it is informative to consider also nonlinear relationships between aggregates. We thus repeat the parametric regression at the industry and sector levels. In the next section we ask whether our estimated firm level hiring rules are consistent with those regressions.

Table 7: Aggregate employment asymmetry

$$\begin{aligned} \text{Regressions (Ia/b, III): } & n_t^i = \beta_0 + \beta^i + \beta_4 z + \beta_5 z^2 \\ \text{Regressions (IIa/b, IV): } & n_t^i = \beta_0 + \beta^i + \beta_4 z + \beta_7 z \mathbb{I}\{z < 0\} \end{aligned}$$

Explanatory variables	Industry regressions				Sectoral regression	
	(Ia)	(Ib)	(IIa)	(IIb)	(III)	(IV)
	quadratic		piece-wise lin.		quadr.	piece-wise lin.
β_4	0.582*** (0.0220)	0.686*** (0.0228)	0.389*** (0.0416)	0.430*** (0.0431)	1.333*** (0.156)	0.757** (0.367)
β_5	-1.370*** (0.223)	-1.648*** (0.232)			-13.63*** (3.281)	
β_7			0.380*** (0.0677)	0.494*** (0.0706)		1.291** (0.596)
N	4,375	4,375	4,375	4,375	51	51
N industries	86	86	86	86	-/-	-/-
R^2	0.219	0.258	0.218	0.258	0.562	0.553
Weighting	no	yes	no	yes	no	no
Pos. Resp.	2.28%	2.68%	1.69%	1.88%	2.06%	1.46%
Neg. Resp.	-2.80%	-3.31%	-3.35%	-4.03%	-3.07%	-3.94%
Diff. sign.?	Yes***	Yes***	Yes***	Yes***	Yes***	Yes***

Note: *, **, *** significantly different from 0 at the 10%, 5%, 1% level, respectively. This table displays regression results using data from the NBER-CES manufacturing database 1958-2009. Industry-level regressions, columns (I-II), are at the level of four-digit NAICS industries, sectoral regressions, columns (III-IV), at the level of the manufacturing sector.

Table 7 shows that concavity does not “wash out,” but is also present at higher levels of aggregation. At the firm level, the typical employment response after a negative TFP shock was about 1.5 times as strong as that to a positive TFP shock. The results in Table 7 show that firing relative

to hiring at the level of four-digit NAICS industries ranges between 1.3 and 2 depending on the specification. The same is true once we repeat the analysis at the sectoral level. In general, piecewise linear hiring rules suggest stronger asymmetries than linear-quadratic rules. This is probably driven by outliers in the data which have a stronger impact in a linear setup.

5.3 Quantitative significance

In this section we compute the effect of concave hiring rules on employment growth under the assumption that TFP shocks are the only source of uncertainty. While we cannot expect to generate all the volatility observed in the data, we can make the case that our mechanism has an important quantitative contribution. The computations thus make a quantitative connection between the concave hiring rules estimated in Section 4 and the moments of employment growth measured in Section 3. They complement the qualitative connection provided by the propositions in Section 2.

Notation follows the general setup of Section 2. We assume throughout this section that TFP innovations are observed without noise, that is, firms respond to the signal $s_t^i = u_t^a + u_t^i$, where the aggregate and idiosyncratic TFP innovations are mutually uncorrelated and normally distributed as $u_t^a \sim \mathcal{N}(0, \sigma_a^2)$ and $u_t^i \sim \mathcal{N}(0, \sigma_i^2)$, respectively. A symmetric distribution for TFP shocks is motivated by the evidence in Section 4. The estimated volatilities are $\sigma_a = 0.041$ and $\sigma_i = 0.174$.

We work with the linear-quadratic hiring function

$$n_t^i = f(s_t^i) = \alpha s_t^i - \beta (s_t^i)^2 \quad (14)$$

where $\alpha = 0.0827$ and $\beta = 0.0998$ are the estimated response coefficients taken from Specification (I) in Table 6. We compute model implied moments of the cross-sectional and time series distributions of employment growth $f(s_t^i)$ by feeding a simulated sequence of signals through this concave hiring function. Table 8 reports relevant statistics, together with comparison numbers taken from the data as well as a statistics that would obtain for a linear hiring function. We discuss these statistics in detail below.

Appendix C.5 redoes the calculations of this section under various alternative specifications. In particular, we show that the benchmark specification produces statistics that are more conservative than alternatives in which we add noise to the signal, as in formula (6). The additional noise increases the volatility of the firm-specific signals, which produces stronger effects through the concavity of the hiring rule and thus results in statistics that are larger in absolute value.²⁶

We have explored two alternative sets of statistics that we do not report. First, we have considered the non-parametric hiring rule displayed in Figure 8. The quantitative effects using that rule are slightly stronger than the ones obtained using the parametric hiring rule. We focus here on the parametric version because it allows for a more transparent analysis given that we can derive closed form expressions for many relevant moments. Second, we have studied a hiring rule

²⁶We have also explored an alternative piece-wise linear function, as estimated for Specification (III) in Table 6, for which we also find stronger quantitative significance. Appendix C.5 details these various alternative specifications.

that is not concave, but instead linear with an inaction region around zero. The resulting statistics resemble those implied by the linear function. This is not surprising in light of our propositions: the concavity of the function around the negative signal threshold for inaction is counterbalanced by the convexity around the positive threshold.

Table 8: Moments of employment growth with different shocks

<i>A. Cross-sectional</i>				
	(I)	(II)	(III)	
<i>Moments</i>	$\frac{IQR(n^i u^a=-\sigma_a)}{IQR(n^i u^a=+\sigma_a)}$	$\gamma(n^i u^a=0)$	$k(n^i u^a=0)$	
Data	1.267	-0.477	-0.046	
Linear Hiring	1	0	0	
Concave Hiring	1.220	-1.177	-0.269	
<i>B. Time-series</i>				
	(IV)	(V)	(VI)	(VII)
<i>Moments</i>	Firm skewn.	Aggr. skewn.	$\frac{\bar{n}(u^a=0)-\bar{n}(u^a=-\sigma_a)}{\alpha\sigma_a}$	$\frac{\bar{n}(u^a=+\sigma_a)-\bar{n}(u^a=0)}{\alpha\sigma_a}$
Data	-0.386	-0.937	1.103	0.899
Linear Hiring	0	0	1	1
Concave Hiring	-1.037	-0.244	1.049	0.950

Note: Moments of the employment growth distribution are defined in equations (15) for column (I), (16) for column (II), (17) for column (III) and (18) for column (IV). Aggregate employment, used in columns (V)-(VII), is defined in equation (19).

Countercyclical cross-sectional dispersion

Column (I) of Table 8 shows that concave hiring rules are an important source of countercyclical dispersion even if the only shocks are symmetric TFP shocks. We consider the cyclical movement of the interquartile range. In the first row – labeled “Data” – we report the ratio of the interquartile range in NBER recessions versus booms: dispersion is 27% higher in recessions (see Table 1). If firm’s hiring rules were linear, then symmetry and homoskedasticity of shocks would imply a constant IQR over the business cycle, as indicated by a ratio of one in the second row.

In contrast, with a concave hiring rule the IQR depends on the realization u_t^a . In fact, there is a closed form solution for the IQR given u^a :

$$IQR(n_t^i|u_t^a) \equiv (2\alpha - 4\beta u_t^a)\sigma_i\Phi^{-1}(0.75) \quad (15)$$

where Φ is the standard normal cdf. The IQR also depends on the degree of concavity in the hiring function here measured by β (relative to α), as well as the idiosyncratic dispersion of shocks σ_i .

The third row reports the ratio of the IQR after a negative one standard deviation shock to aggregate TFP versus a positive one standard deviation shock. A concave hiring rule induces a lot of countercyclical volatility: feeding aggregate shocks alone increases the IQR in “bad times” by 22%. These numbers illustrate that our estimated concavity is sufficiently strong to generate the same order of magnitude of fluctuations in dispersion as in the data.

Negative skewness at the firm level

Columns (II)-(IV) of Table 8 show that concave hiring rules together with symmetric TFP shocks generate significant negative skewness in individual firm employment growth, both in the cross section and the time series. The first row in columns (II)-(III) reports the the long-run average coefficient of skewness and Kelley skewness for the cross section of firms in the data (again, see Table 1). With a linear hiring rule ($\beta = 0$), employment growth would not be skewed, so both numbers would be zero, as reported in the second row. This is because the cross-sectional distribution of shocks is always symmetric and its linear transformation will also be symmetric.

Our setup again allows closed form solution for both measures. At the mean aggregate innovation $u^a = 0$, we have

$$\gamma(n_t^i | u_t^a = 0) = -\frac{6\alpha^2\beta\sigma_i^4 + 8\beta^3\sigma_i^6}{[\alpha^2\sigma_i^2 + 2\beta^2\sigma_i^4]^{3/2}} \quad (16)$$

$$k(n_t^i | u_t^a = 0) = -\frac{\beta}{\alpha}\sigma_i\Phi^{-1}(0.9). \quad (17)$$

At our estimates, the coefficient of skewness, defined in equation (16) is equal to -1.177 , while Kelley skewness, $k(n_t^i | u_t^a = 0)$ as defined in equation (17), equals -0.269 . Both numbers are thus somewhat higher than the conditional skewness in the data, suggesting that additional idiosyncratic shocks remove some skewness.

Column (IV) shows that concave hiring rules also induce significant skewness at the firm level in the time series. The first row reports the cross sectional average of individual firms' sample coefficient of skewness for our balanced panel of firms. We compute time-series skewness with the balanced panel since the simple model here does not consider entry and exit, a factor that weakens negative skewness for the full panel reported in Table 4. As reported in the second row, a linear hiring rule again delivers zero skewness: since the overall signal has a symmetric distribution centered at zero, the linear hiring rule will then produce an employment growth distribution that is similarly symmetric and centered at zero.

To evaluate negative skewness predicted by our mechanism, we simulate a panel of firms that resembles the data panel we document in Table 4: $N = 1,900$ firms are simulated over $T = 37$ years 10,000 times. As for the actual data panel, we then calculate, for each simulated panel, the cross sectional average of the coefficients of skewness for the firm-level sample of n_t^i , where each firm has a resulting mean \bar{n}^i and volatility Vol^i :

$$Asym = \frac{1}{N} \sum_{i=1}^N \frac{1}{T} \sum_{t=1}^T \frac{(n_t^i - \bar{n}^i)^3}{(Vol^i)^{3/2}}. \quad (18)$$

The third row in column (IV) reports the average of these statistics across all simulations. We obtain skewness that is slightly more negative than in the data.

Aggregate employment growth

Columns (V-VII) of Table 8 show that concave hiring rules not only affect firm level moments, but also moments at higher levels of aggregation. In particular, they generate significant negative skewness for aggregate employment growth. Moreover, relative to the case of linear hiring rules, they amplify the effect of aggregate TFP shocks on employment growth, and can reproduce a significant share of the concavity in the aggregate relationships between TFP and employment growth documented in Subsection 5.2.

Column (V) looks at aggregate skewness. The row labeled “Data” considers average time series skewness at the NAICS-4 manufacturing industry level reported in Table 4. To obtain a model counterpart, we integrate over the individual decision rules from (14)²⁷ to obtain average employment growth

$$\bar{n}(u^a) = \frac{1}{N} \sum_{i=1}^N n_t^i = \alpha u^a - \beta [(u^a)^2 + \sigma_i^2]. \quad (19)$$

We simulate aggregate shocks u_t^a for $T = 37$ years, compute the sample coefficient of skewness and report in column (V) the average coefficient over 10,000 samples.

As before, a purely linear rule implies zero skewness since the aggregate shocks are also symmetric. In contrast, the estimated concave hiring function leads to negative skewness since the cross-sectional average action responds stronger to negative innovations. Compared to the average time-series skewness at the firm level, the aggregate one is not so negative. This happens because aggregate shocks are smaller and thus the relevant region for the average action features a less pronounced concavity.

To assess asymmetric amplification of aggregate shocks, we define a summary measure that can be computed both from the data and from the model. We consider the movement in aggregate employment associated with an aggregate TFP innovation and divide it by a scaled version of the innovation itself:

$$\frac{\bar{n}(u^a) - \bar{n}(0)}{\alpha u^a} = 1 - \frac{\beta u^a}{\alpha}. \quad (20)$$

For any given level of u^a , this ratio can be computed from the aggregate data by using the fitted value from the industry level regression (Ia) in Table 7. It can also be derived from our firm level model using implied aggregate employment growth (19). For both model and data measures we average across industries.

Intuitively, the ratio thus works like an elasticity of employment growth with respect to TFP shocks – it shows the amplification (dampening) generated by concave hiring rules for bad (good) shocks. With a linear hiring rule we have $\beta = 0$ and the ratio (20) is equal to one – the response of employment growth is proportional to the TFP shock. More generally, a negative (positive) realization of u^a generates a larger (smaller) response. The magnitude of the effect further depends on the relative strength of concavity $\frac{\beta}{\alpha}$.

²⁷We assume that distribution of lagged employment levels is orthogonal to the shock distribution; then aggregate employment growth equals average employment growth.

In columns (VI) and (VII), we report the ratio for $u^a = -\sigma_a$ and $u^a = +\sigma_a$, respectively. In the data, the asymmetric relationship between TFP and employment growth implies an amplification by 10% in bad times and a dampening by 10% in good times.²⁸ Aggregating the effect of our estimated firm level hiring rules, we get an amplification of 1.05 and a dampening of 0.95, respectively. Our mechanism thus produces about half of the asymmetric response at the industry level.

5.4 Concave hiring rules and volatility shocks

In this section we study the interaction of concave hiring rules and shocks to the cross sectional volatility of TFP. Recent literature following Bloom (2009) explored the role of such shocks as drivers of the business cycle. We look at both volatility shocks in isolation and at correlated shocks that combine a drop in average TFP with an increase in cross sectional volatility, as is common in applications (for example, Bloom et al. (2012); Schaal (2012); Vavra (2014); Arellano et al. (2010)).

In order to assess volatility shocks, we calibrate the cross-sectional dispersion of idiosyncratic TFP shocks separately for booms and recessions. In our data, the average interquartile range in NBER recessions is about 7% larger than in booms – this is our baseline number for volatility shocks. It is slightly smaller than that in Bloom et al. (2012) who consider only a sample of long-lived establishments and find a 13% higher IQR in recessions.²⁹ For robustness, we also use that number in our exercises, as described below.

We repeat all exercises reported in Table 8 by replacing average TFP shocks with three new types of shocks that change volatility. In the first two exercises, there are no average TFP shocks but only volatility shocks. To connect the model to the data in Table 8, a recession here is defined as a period of a high idiosyncratic volatility. In particular, in the first exercise $\sigma_{i,t}$ moves randomly with equal probability between a high value of 0.18 and a low value of 0.168, so that in a recession, as defined in this exercise, the idiosyncratic risk is 7% larger. In the second exercise, which we refer to as “large” volatility shocks, we make that increase equal to 13%, consistent with Bloom et al. (2012). In the third exercise, which we refer to as “correlated shocks,” we bring back the aggregate TFP shocks that we have used in Table 8, but now make them determine the volatility movements. In particular, whenever there is a negative (positive) u_t^a shock, the idiosyncratic risk $\sigma_{i,t}$ moves to its high (low) value of 0.18 and 0.168, respectively.

Since in these exercises we retain symmetry of TFP shocks, there is no effect on cross sectional skewness: the numbers in Columns (II)-(III) are exactly the same as under with average TFP shock only, regardless of the type of volatility shock used. As long as the hiring rule is linear, firm level time series skewness in Column (IV) changes only with correlated shocks, and even then the effect is quantitatively small.

²⁸In particular, there the negative response of employment growth to a one standard deviation shock equals 0.028. This response is larger by a factor of 1.1 compared to the linear effect given by the linear coefficient 0.582 times the value of the standard deviation of the shock, which equals 0.0436. Similarly, the positive response of 0.0228 is a factor of 0.9 of its linear counterpart.

²⁹See their Table 1, column 1. They also find a higher long-run dispersion of TFP shocks than we do; as shown in Section 4.1 our dispersion is roughly consistent with the one found in Syverson (2004).

With concave hiring rules, however, volatility shocks can contribute significantly to time series skewness at the firm level. Intuitively, a mean preserving spread implies that more firms receive very bad signals to which they respond strongly, while at the same time more firms receive very good signals to which they respond little. The skewness induced by our mechanism is thus reinforced. With baseline volatility shocks only, negative skewness is -1.16 , whereas with large volatility shocks it drops to -1.44 . With correlated shocks, we obtain -1.08 .

With volatility shocks, the distribution of employment growth mechanically reflects counter-cyclical volatility in TFP shocks. With a linear hiring rule, when volatility shocks are the only sources of changes in volatility, the IQR of employment growth thus increases 7% with baseline volatility shocks or 13% with large shocks. Even the latter number is substantially below the 27% in the data reported in column (I) of Table 8. As long as volatility shocks are uncorrelated with u^a , they cannot contribute to the moment reported in column (I): equation (15) shows that the cross sectional IQR is proportional to cross sectional volatility.

Interesting interaction between concavity and volatility shocks obtains in the case of correlated shocks. Indeed, the IQR of employment growth now increases by 30.5% in bad times. This increase is notably stronger than the 22% we obtain with average TFP shocks alone. Intuitively, if an average productivity shock moves the distribution of signals into the concave region of the estimated hiring rule, then an additional mean preserving spread makes our mechanism stronger. We conclude that for the shock specification that is most common in applications, there are synergies between concave hiring rules and volatility shocks.

Concave hiring rules also induce first order effects of idiosyncratic volatility on aggregates. Indeed, as long as $\beta > 0$, aggregate employment (19) is decreasing in idiosyncratic uncertainty σ_i . Quantitatively, if we keep the aggregate shock u^a constant at zero, but change idiosyncratic volatility from its long run average to its baseline high value, aggregate employment growth drops *as if* it had been hit by a negative aggregate shock equal to 0.08 standard deviations. Conversely, when the idiosyncratic volatility drops to its low value, aggregate action increases as if it had been shocked by a positive innovation of 0.04 standard deviations.

With correlated shocks, movements in idiosyncratic volatility alter the amplification and dampening of TFP shocks reported in columns (VI) and (VII) of Table 8. Indeed, the ratio (20) increases to 1.12 if evaluated in bad times ($u^a = -\sigma_a$): the amplification of bad TFP shocks more than doubles compared to the case of only average changes. At the same time, the ratio evaluated in good times ($u^a = \sigma_a$) increases to 0.99. Intuitively, with a smaller volatility, most firms receive signals that move their actions closer to the linear function, and thus the dampening effect is smaller.

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A A model with asymmetric information processing

Here we present a simple model based on information processing under ambiguity that delivers asymmetric hiring decision rules. There is a continuum of firms that at the beginning of each period get an idiosyncratic noisy signal about end-of-period productivity z_t^i . After observing the signal, each firm chooses employment in a competitive labor market, where the wage is \bar{w} . At the end of the period, productivity is realized. Thus the firm problem is a repetition of static hiring decisions based on a signal-extraction problem within the period.

Firm i 's log productivity is described as

$$z_t^i = u_t^a + u_t^i - 0.5 (\sigma_a^2 + \sigma_u^2)$$

where u_t^a is an aggregate shock, normally distributed with mean \bar{a} and variance σ_a^2 , and u_t^i is an idiosyncratic, firm-specific shock, normally distributed with mean 0 and variance σ_u^2 . The variances σ_a^2 and σ_u^2 are constant over time and known to the firm. The end-of-period profit is

$$\exp(z_t^i) (L_t^i)^\alpha - \bar{w} L_t^i$$

where L_t^i is employment chosen by firm i . The noisy signal about firm's i productivity is

$$s_t^i = z_t^i + \sigma_e \varepsilon_t^i$$

where ε_t^i is a standard normal innovation, *iid* across time and firms. The key mechanism through which this simple signaling model generates asymmetries is to assume that firms are ambiguous about the information precision of their signals. In particular, similarly to the model used in [Epstein and Schneider \(2008\)](#), we assume that each firm is ambiguous about the value of σ_e and has a set of beliefs given by

$$\sigma_e \in [\underline{\sigma}_e, \bar{\sigma}_e]$$

The objective of the firm is choose L_t^i to maximize the multiple priors utility:

$$\max_{L_t^i} \min_{[\underline{\sigma}_e, \bar{\sigma}_e]} (L_t^i)^\alpha E^{\sigma_e} [\exp(z_t^i) | s_t^i] - \bar{w} L_t^i$$

where the minimization operator reflects the firm's ambiguity-aversion. Faced with uncertainty over the signal-to-noise ratio, the ambiguity-averse firm acts *as if* the worst-case σ_e characterizes the true DGP. The worst-case σ_e minimizes the conditional expectation of end-of-period profits, which are only a function of the expected z_t^i conditional on the observed signal s_t^i . Details and axiomatic foundations for the multiple priors utility are provided in [Gilboa and Schmeidler \(1989\)](#) for the static case and in [Epstein and Schneider \(2003\)](#) for the dynamic version.

Because of the normality of innovations, the problem above is equivalent to

$$\max_{L_t^i} \min_{[\underline{\sigma}_e, \bar{\sigma}_e]} \exp \left[E^{\sigma_e} (z_t^i | s_t^i) + \frac{1}{2} \text{var}^{\sigma_e} (z_t^i | s_t^i) \right] (L_t^i)^\alpha - \bar{w} L_t^i. \quad (21)$$

To characterize the optimal solution, it is analytically helpful to define the relative precision of signal γ_t as a function of a given $\sigma_{e,t}^2$:

$$\gamma_t = \frac{\text{var}(z_t^i)}{\text{var}(z_t^i) + \sigma_{e,t}^2}.$$

After observing the signal, the posterior conditional mean and variance of z_t^i are given by

$$\begin{aligned} E(z_t^i | s_t^i) &= \gamma_t \left[s_t^i + \frac{1}{2} \text{var}(z_t^i) \right] - \frac{1}{2} \text{var}(z_t^i) \\ \text{var}(z_t^i | s_t^i) &= (1 - \gamma_t) \text{var}(z_t^i). \end{aligned}$$

The firm problem in (21) then simplifies to

$$\max_{L_t^i} \min_{[\underline{\sigma}_\varepsilon, \bar{\sigma}_\varepsilon]} \exp(\gamma_t s_t^i) (L_t^i)^\alpha - \bar{w} L_t^i.$$

The solution to this problem results in a hiring policy that is based on the worst case precision γ_t^* characterized by:

$$L_t^i = \left[\frac{\alpha}{\bar{w}} \exp(\gamma_t^* s_t^i) \right]^{\frac{1}{1-\alpha}}; \quad \gamma_t^* = \begin{cases} \bar{\gamma} & \text{if } s_t^i < 0 \\ \underline{\gamma} & \text{if } s_t^i \geq 0. \end{cases} \quad (22)$$

The interpretation of the optimal solution is that the firm acts *as if* the signal precision is high for bad news and low for good news. The employment decision is then to maximize expected profits under the worst-case precision γ_t^* . This results in an asymmetric hiring decision rule such that the firm that receives a negative signal $s_t^i = -x$ contracts its employment by more than it would expand it if the firm would have received a positive signal of the same magnitude, $s_t^i = x$.

B Proofs for Section 2

Proof of Proposition 1. *Part 1.* Write the derivative of aggregate employment growth at the point a as

$$\frac{d}{d\bar{a}} E[f(s)]|_{\bar{a}=a} = \frac{d}{d\bar{a}} \int f(\epsilon + \bar{a}) g(\epsilon) d\epsilon |_{\bar{a}=a} = \int f'(\epsilon + a) g(\epsilon) d\epsilon.$$

Since $f'' < 0$, we have that for every realization of ϵ ,

$$a' > a \Rightarrow f'(a' + \epsilon) < f'(a + \epsilon).$$

Part 2. Define the function

$$h(a) := \text{var}(f(s) | a) = E[f(s)^2 | a] - E[f(s) | a]^2.$$

Then we have

$$\begin{aligned} h'(a) &= 2(E[f(s) f'(s) | a] - E[f(s) | a] E[f'(s) | a]) \\ &= 2 \text{cov}(f(s), f'(s) | a) \end{aligned}$$

which is negative for all a since f is strictly increasing in s and f' is strictly decreasing in s .

Part 3. The conditional cdf of employment growth at some point \bar{f} can be written as

$$\begin{aligned} G_f(\bar{f} | a) &= \Pr(f(a + \epsilon) \leq \bar{f}) \\ &= \Pr(a + \epsilon \leq f^{-1}(\bar{f})) \\ &= G(f^{-1}(\bar{f}) - a). \end{aligned}$$

The inverse conditional cdf is therefore

$$G_f^{-1}(\bar{x}|a) = f(a + G^{-1}(\bar{x})).$$

An increase in a now means shifting the pair of points $a + G^{-1}(\underline{x})$ and $a + G^{-1}(\bar{x})$ at which we evaluate f by the same amount; concavity then means the interquantile range shrinks. ■

Proof of Proposition 2. The proof uses the following result (Theorem 3.1 in van de Geer and Wegkamp (2011)).

Lemma. Let x denote a random variable, ϕ denote a nonconstant convex function, $\mu_j(y)$ denote the j th centered moment and $\sigma(y)$ the standard deviation of a random variable y . Then for all $k = 1, 2, \dots$, provided the moments exist, we have

$$\frac{\mu_{2k+1}(x)}{\sigma^{2k+1}(x)} \leq \frac{\mu_{2k+1}(\phi(x))}{\sigma^{2k+1}(\phi(x))}.$$

Part 1.a. Define $\phi(x) = f^{-1}(x) - a$. If x represents the distribution of s , then $\phi(x)$ represents the distribution of ε . The function ϕ is convex by concavity of f , and the result follows directly from the lemma.

Part 1.b. Define the ratio of 50-10 interquantile to 90-50 interquantile to be:

$$R(x) = \frac{P_{50}(x) - P_{10}(x)}{P_{90}(x) - P_{50}(x)},$$

where P is the percentile of the distribution. Then the Kelley skewness $k(x)$ is decreasing in $R(x)$ since

$$k(x) = \frac{(P_{90} - P_{50}) - (P_{50} - P_{10})}{(P_{90} - P_{50}) + (P_{50} - P_{10})} = \frac{1 - R(x)}{1 + R(x)} \quad (23)$$

Define the ratios of 50-10 interquantile to 90-50 interquantile for s and f , respectively, to be R_s and R_f . The \bar{x} quantile for s is

$$G_s^{-1}(\bar{x}) = a + G^{-1}(\bar{x}).$$

It follows that

$$\begin{aligned} R_s &= \frac{G^{-1}(50) - G^{-1}(10)}{G^{-1}(90) - G^{-1}(50)} \\ R_f &= \frac{f(a + G^{-1}(50)) - f(a + G^{-1}(10))}{f(a + G^{-1}(90)) - f(a + G^{-1}(50))} \end{aligned}$$

Comparing R_s to R_f , we find:

$$\begin{aligned} \frac{R_f}{R_s} &= \frac{f(a + G^{-1}(50)) - f(a + G^{-1}(10))}{(a + G^{-1}(50)) - (a + G^{-1}(10))} \div \frac{f(a + G^{-1}(90)) - f(a + G^{-1}(50))}{(a + G^{-1}(90)) - (a + G^{-1}(50))} \\ &= f'(\xi_L) \div f'(\xi_R), \quad \xi_L \in (a + G^{-1}(10), a + G^{-1}(50)), \xi_R \in (a + G^{-1}(50), a + G^{-1}(90)) \end{aligned}$$

where the last equality comes from Mean Value Theorem. Since f' is decreasing and $\xi_L < \xi_R$, it follows that $R_f > R_s$. By equation (23) we have $k(n|a) < k(s|a)$.

Part 2.a: Write aggregate employment growth as a function of aggregate news

$$g(a) = E[f(a + \varepsilon) | a]. \quad (24)$$

The function g is also concave. Indeed, for any $a \neq a'$,

$$\begin{aligned} g(\lambda a + (1 - \lambda) a') &= E[f(\lambda a + (1 - \lambda) a' + \varepsilon) | \lambda a + (1 - \lambda) a'] \\ &= E[f(\lambda(a + \varepsilon) + (1 - \lambda)(a' + \varepsilon)) | \lambda a + (1 - \lambda) a'] \\ &> \lambda E[f(a + \varepsilon) | \lambda a + (1 - \lambda) a'] + (1 - \lambda) E[f(a' + \varepsilon) | \lambda a + (1 - \lambda) a'] \\ &= \lambda E[f(a + \varepsilon) | a] + (1 - \lambda) E[f(a' + \varepsilon) | a'] \\ &= \lambda g(a) + (1 - \lambda) g(a') \end{aligned}$$

where the last line uses the fact that ε is independent of a .

Define now $\phi(x) = g^{-1}(x)$. If x represents the distribution of aggregate employment growth $E[f(a + \varepsilon) | a]$, then $\phi(x)$ represents the distribution of a . Since ϕ is convex, the result follows directly from the lemma.

Part 2.b. Denote by $G_a(\bar{a})$ the CDF function for random variable a , and G_g CDF function for random variable $g = g(a)$, where $g(a)$ is defined in equation (24). We have

$$G_g(\bar{g}) = \Pr[g(a) \leq \bar{g}] = G_a(g^{-1}(\bar{g}))$$

In a similar fashion of Proposition 1, Part 3, we can express the \bar{x} percentile of g , call it $\bar{g}_{\bar{x}}$, as:

$$\bar{g}_{\bar{x}} = g[G_a^{-1}(\bar{x})].$$

Now we are ready compare the ratios of 50-10 interquartile to 90-50 interquartile for g and a . First we write the ratio for a as

$$R_a = \frac{G_a^{-1}(50) - G_a^{-1}(10)}{G_a^{-1}(90) - G_a^{-1}(50)},$$

second we write the ratio for g as:

$$R_g = \frac{G_g^{-1}(50) - G_g^{-1}(10)}{G_g^{-1}(90) - G_g^{-1}(50)} = \frac{g[G_a^{-1}(50)] - g[G_a^{-1}(10)]}{g[G_a^{-1}(90)] - g[G_a^{-1}(50)]}$$

Comparing R_g to R_a , we find

$$\begin{aligned} \frac{R_g}{R_a} &= \frac{g(G_a^{-1}(50)) - g(G_a^{-1}(10))}{(G_a^{-1}(50)) - (G_a^{-1}(10))} \div \frac{g(G_a^{-1}(90)) - g(G_a^{-1}(50))}{(G_a^{-1}(90)) - (G_a^{-1}(50))} \\ &= g'(\xi_L) \div g'(\xi_R), \quad \xi_L \in (G_a^{-1}(10), G_a^{-1}(50)), \xi_R \in (G_a^{-1}(50), G_a^{-1}(90)) \end{aligned}$$

where the last equality comes from Mean Value Theorem. We have proved in Part 2.a. that g is strictly concave, and therefore g' is decreasing. Since $\xi_L < \xi_R$, we obtain that $R_g > R_a$. By equation (23), it follows that the Kelley skewness for g is smaller than that of a .

Proof of Proposition 3.

Part 1. Define $y = f(a' + \varepsilon)$ and $x = f(a + \varepsilon)$. The random variable x represents the cross sectional distribution at a , whereas the random variable y represents the cross sectional distribution

at a' . Since f is strictly increasing, we can write $y = \phi(x)$, where

$$\phi(x) = f(a' - a + f^{-1}(x)).$$

By the lemma in the proof of Proposition 2, skewness is higher at a' if ϕ is convex. The first and second derivatives are

$$\begin{aligned}\phi'(x) &= \frac{f'(a' - a + f^{-1}(x))}{f'(f^{-1}(x))} > 0 \\ \phi''(x) &= \frac{f'(f^{-1}(x)) f''(a' - a + f^{-1}(x)) - f'(a' - a + f^{-1}(x)) f''(f^{-1}(x))}{f'(f^{-1}(x))^3}.\end{aligned}$$

Since $f' > 0$ and $f'' < 0$, we have $\phi''(x) > 0$ if and only if

$$-\frac{f''(a' - a + f^{-1}(x))}{f'(a' - a + f^{-1}(x))} < -\frac{f''(f^{-1}(x))}{f'(f^{-1}(x))}.$$

This is true if absolute risk aversion is decreasing everywhere. The relationship between increasing absolute risk aversion and lower skewness follows by reversing the inequalities.

Part 2. Taking derivative of R_f , defined above in the proof of Proposition 2, *Part 1.b*, with respect to a :

$$\begin{aligned}\frac{d}{da} R_f &= \frac{[f'(a + G^{-1}(50)) - f'(a + G^{-1}(10))] [f(a + G^{-1}(90)) - f(a + G^{-1}(50))]}{[f(a + G^{-1}(90)) - f(a + G^{-1}(50))]^2} \\ &\quad - \frac{[f(a + G^{-1}(50)) - f(a + G^{-1}(10))] [f'(a + G^{-1}(90)) - f'(a + G^{-1}(50))]}{[f(a + G^{-1}(90)) - f(a + G^{-1}(50))]^2}\end{aligned}$$

To determine the sign of this derivative we only need to focus on the sign the numerator term. Dividing the numerator term by a positive term

$$[G^{-1}(50) - G^{-1}(10)] [G^{-1}(90) - G^{-1}(50)]$$

and applying Mean Value Theorem, we obtain the term

$$f''(\xi_L) f'(\xi_R) - f'(\xi_L) f''(\xi_R) \tag{25}$$

where

$$\xi_L \in (a + G^{-1}(10), a + G^{-1}(50)); \quad \xi_R \in (a + G^{-1}(50), a + G^{-1}(90)).$$

Divide the term in (25) by $f'(\xi_L) f'(\xi_R)$, which is positive, to obtain

$$\frac{f''(\xi_L)}{f'(\xi_L)} - \frac{f''(\xi_R)}{f'(\xi_R)}$$

If $-f''(s)/f'(s)$ is decreasing in s then R_f is decreasing in a . As a consequence, the Kelley skewness is increasing in a . The relationship between increasing absolute risk aversion and lower Kelley skewness follows by reversing the inequalities.

C Robustness

C.1 Additional empirical evidence

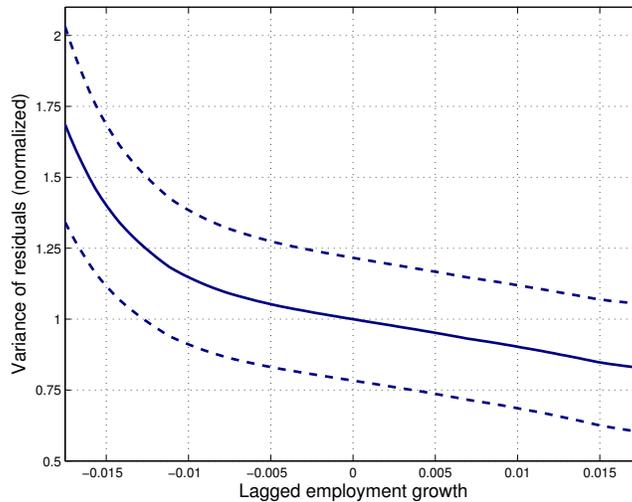
C.1.1 Conditional heteroskedasticity in aggregate employment

We follow [Bachmann et al. \(2013\)](#) and analyze conditional heteroskedasticity in aggregate employment growth. To that end, we use monthly data on employment in the private economy collected by BLS 1947:Jan-2015:Dec, aggregate to quarterly frequency and estimate the following model:

$$\Delta \log E_t = c + \beta_0 t + \sum_{i=1}^p \rho^i \Delta \log E_{t-i} + \varepsilon_t \quad (26)$$

where $\varepsilon_t \sim N(0, \sigma_t)$ and σ_t varies over time. We first determine the optimal lag length p to be 11 using the Akaike information criterion, then estimate (26) by OLS, recover the residuals ε_t , square them and regress them non-parametrically on lagged aggregate employment growth $\Delta \log E_{t-1}$. To better illustrate the volatility changes of aggregate employment depending on the level of employment growth, we have normalized the variance at unity for $\Delta \log E_{t-1} = 0$. As [Figure 10](#) shows, volatility is about twice as high for aggregate employment growth rates of -1.8% than for +1.8%. These differences are statistically significant in the sense the 95% error bands at either end do not overlap.

Figure 10: Conditional heteroskedasticity of aggregate employment growth



Note: We follow the procedure in [Bachmann et al. \(2013\)](#) to estimate how conditional aggregate volatility of employment growth depends on the level of employment growth.

C.1.2 Employment patterns by sector

In [Sections 3.2](#) and [3.3](#) we documented employment patterns in the manufacturing sector using the Annual Survey of Manufactures (ASM). The Longitudinal Business Database (LBD) contains

Table 9: Employment moments by sector

Moment	Sector								
	Agri- culture	Mining & Logging	Con- struction	Manu- facturing	Trspt. & Utilities	Wholesale Trade	Retail Trade	FIRE	Services
<i>A.1 Cross-sectional dispersion</i>									
Long-run average	0.4136	0.3768	0.4538	0.3631	0.3800	0.3544	0.3635	0.3520	0.3708
Volatility	0.0110	0.0244	0.0140	0.0105	0.0121	0.0127	0.0124	0.0166	0.0106
$Corr(dE_t^{agg}, \dots)$	-0.0317	-0.3287	-0.4600	-0.5891	-0.2134	-0.2092	-0.1503	-0.1814	-0.2104
<i>A.2 Cross-sectional skewness</i>									
Long-run average	-0.0264	-0.0145	-0.0360	-0.0712	-0.0528	-0.0816	-0.0604	-0.0605	-0.0370
Volatility	0.0983	0.1595	0.0732	0.1039	0.1071	0.0991	0.0796	0.1490	0.0897
$Corr(dE_t^{agg}, \dots)$	0.3808	0.6035	0.5139	0.6400	0.5529	0.5391	0.4483	0.5716	0.5067
<i>B.1 Longitudinal volatility – firm level</i>									
Long-run average	0.3219	0.2918	0.3749	0.2652	0.2732	0.2589	0.2835	0.2580	0.2775
Volatility	0.0079	0.0121	0.0073	0.0065	0.0058	0.0050	0.0044	0.0095	0.0049
$Corr(dE_t^{agg}, \dots)$	0.2318	-0.3027	-0.2147	-0.4565	0.0087	0.0920	0.1157	0.2619	0.0699
<i>B.2 Longitudinal asymmetry – firm level</i>									
Long-run average	-0.0198	-0.0384	-0.0205	-0.0618	-0.0253	-0.0430	-0.0298	-0.0091	-0.0047
Volatility	0.0194	0.0465	0.0361	0.0328	0.0191	0.0180	0.0197	0.0303	0.0169
$Corr(dE_t^{agg}, \dots)$	0.0822	-0.0271	0.5507	0.6200	0.2493	0.3734	0.4318	0.1863	0.0818
<i>C.1 Longitudinal volatility – aggregate level</i>									
Long-run average	0.0497	0.0494	0.0478	0.0298	0.0200	0.0199	0.0161	0.0151	0.0123
Volatility	0.0364	0.0249	0.0253	0.0165	0.0092	0.0078	0.0068	0.0051	0.0056
$Corr(dE_t^{agg}, \dots)$	0.0431	-0.1940	-0.2416	-0.3175	-0.3032	-0.3455	-0.3248	0.0897	-0.3525
<i>C.2 Longitudinal asymmetry – aggregate level</i>									
Long-run average	-0.0310	-0.2566	-0.3029	-0.4804	-0.4126	-0.1758	-0.2488	-0.2824	-0.4525
Volatility	0.8730	0.6651	0.5912	0.6162	0.5855	0.6287	0.6718	0.4597	0.5089
$Corr(dE_t^{agg}, \dots)$	0.0819	0.2451	-0.1401	-0.0344	-0.1074	-0.0129	-0.0389	-0.1282	-0.0389

Figure 11: Cross-firm dispersion and skewness by sector (LBD)

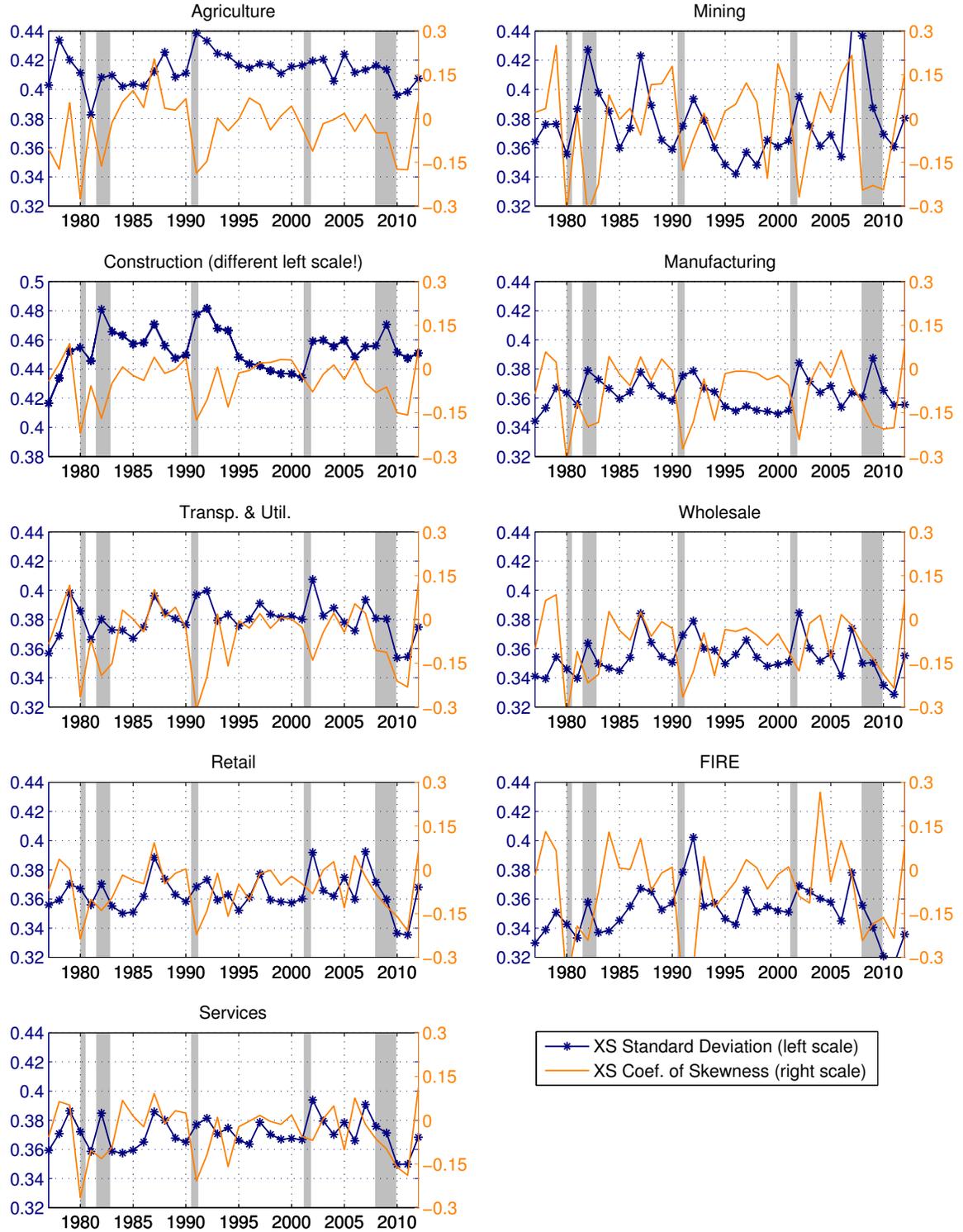


Figure 12: Time-series volatility and asymmetry of firm-level employment growth by sector (LBD)

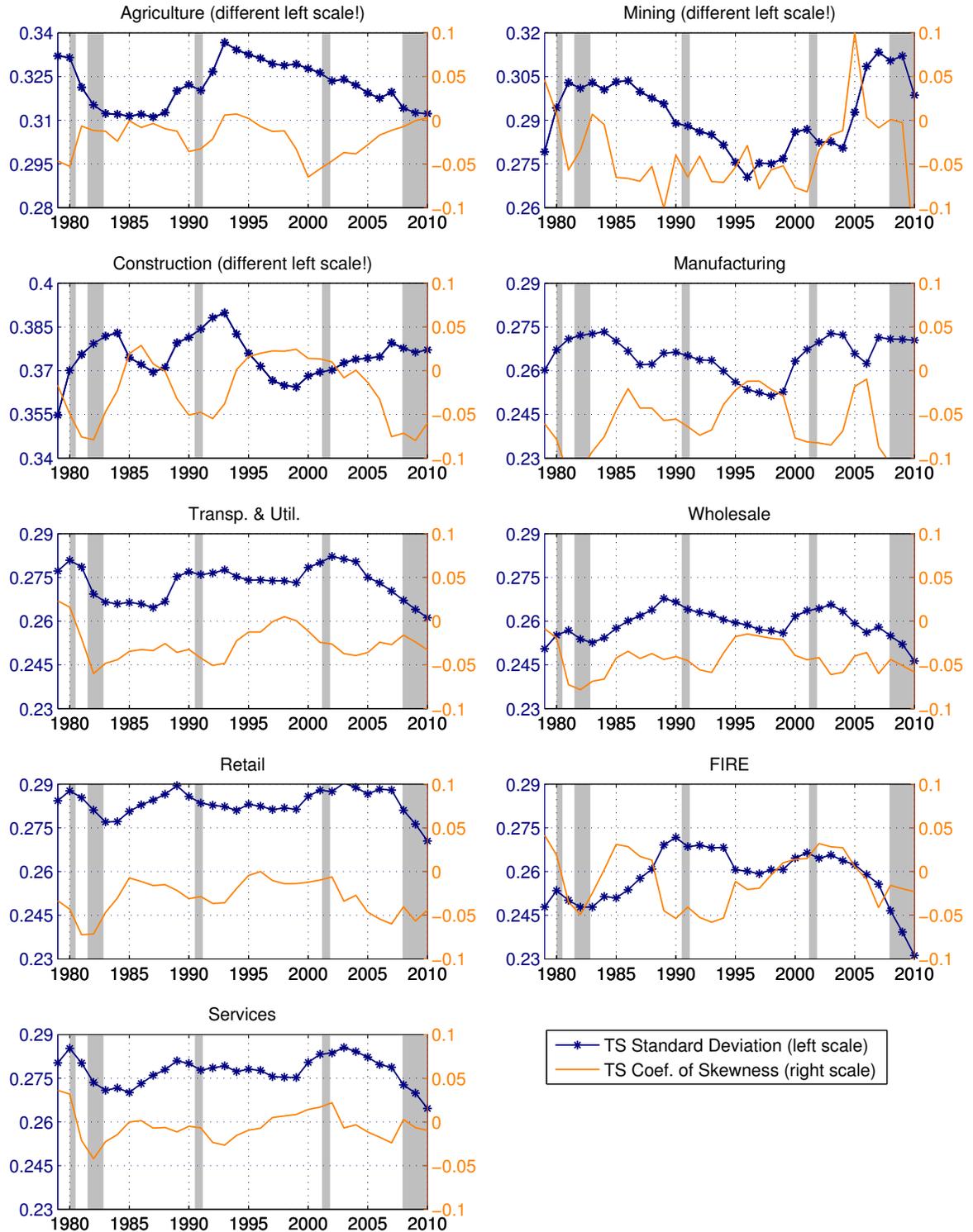
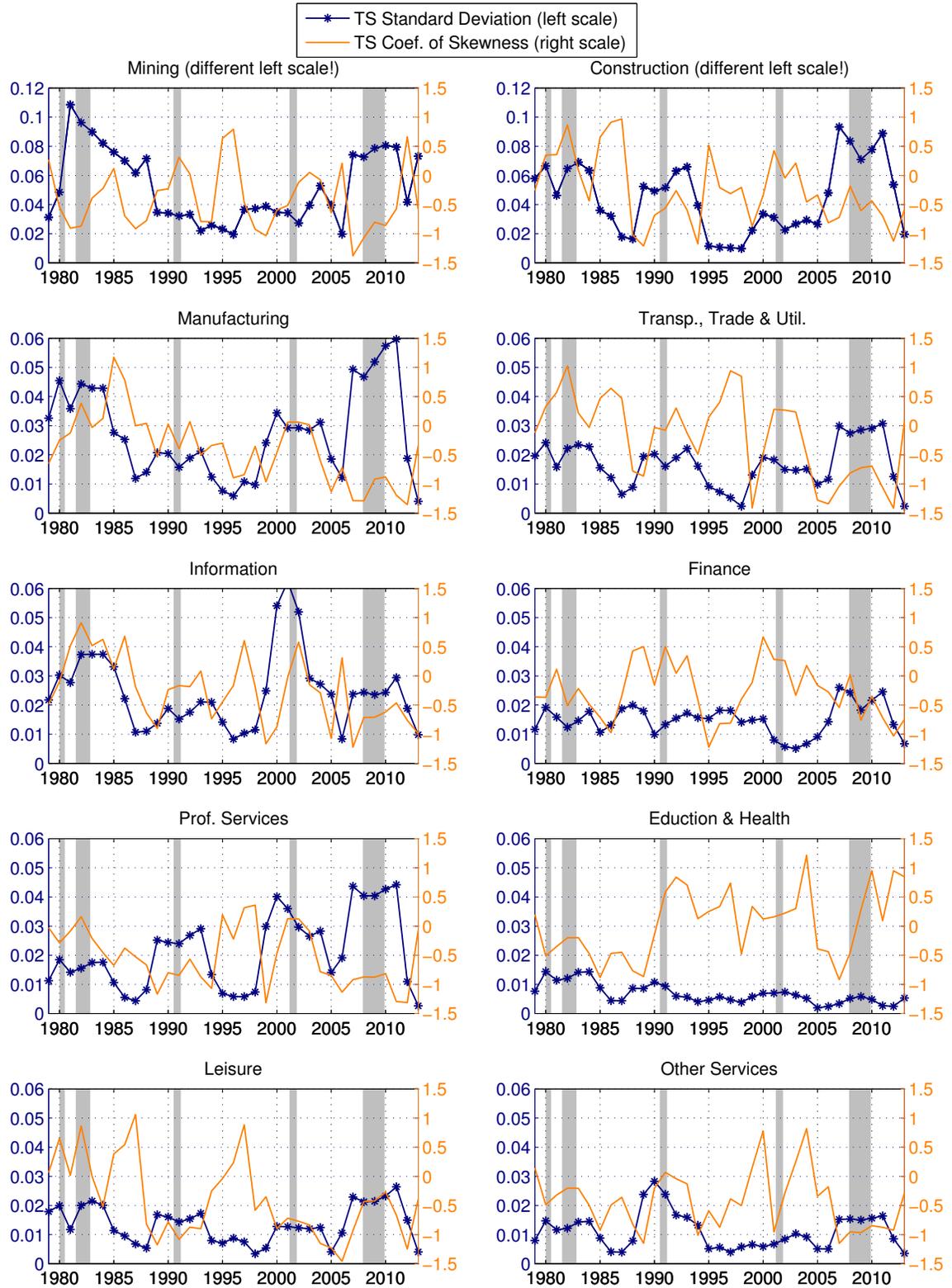


Figure 13: Time-series volatility and asymmetry of aggregate employment growth by sector



employee data for all establishments in the entire economy. Even though it covers the manufacturing (and other establishments) in its entirety, we had preferred the ASM in the main body of our paper because it allowed us to link employment growth to technology shocks, an object which we cannot compute in the LBD, and because it measures employment more precisely. We observe quarterly production employment and the March snapshot of non-production employment. In the LBD, in contrast, we only observe the March snapshots of all employment which thus misses some high-frequency movements.

Even though the ASM provides the more high quality data, we will now use the LBD to document the cross-sectional and time-series patterns of employment growth for each of the ten major sectors in the LBD. We repeat the exercise in Section 3.3 at the sectoral level to check if sectors outside manufacturing behave similarly like manufacturing. We use monthly employment data from BLS 1947-2015 which gives us a longer time horizon and broader scope of employment data. Table 9 displays the time-series patterns of employment growth for the ten major sectors in the private U.S. economy. Interestingly, aggregate employment growth in almost all other sectors exhibit the same patterns as in the manufacturing sector. Though most service sectors are less volatile on average, their volatility is also countercyclical. The only exception to this rule is the Finance sector: the general time series volatility of employment growth is almost one order of magnitude smaller and it is not cyclical, not even with respect to its own sectoral employment. Furthermore, the sectoral employment growth time series are negatively skewed, that is, employment downturns are faster than employment upturns.

C.1.3 Dynamic correlations

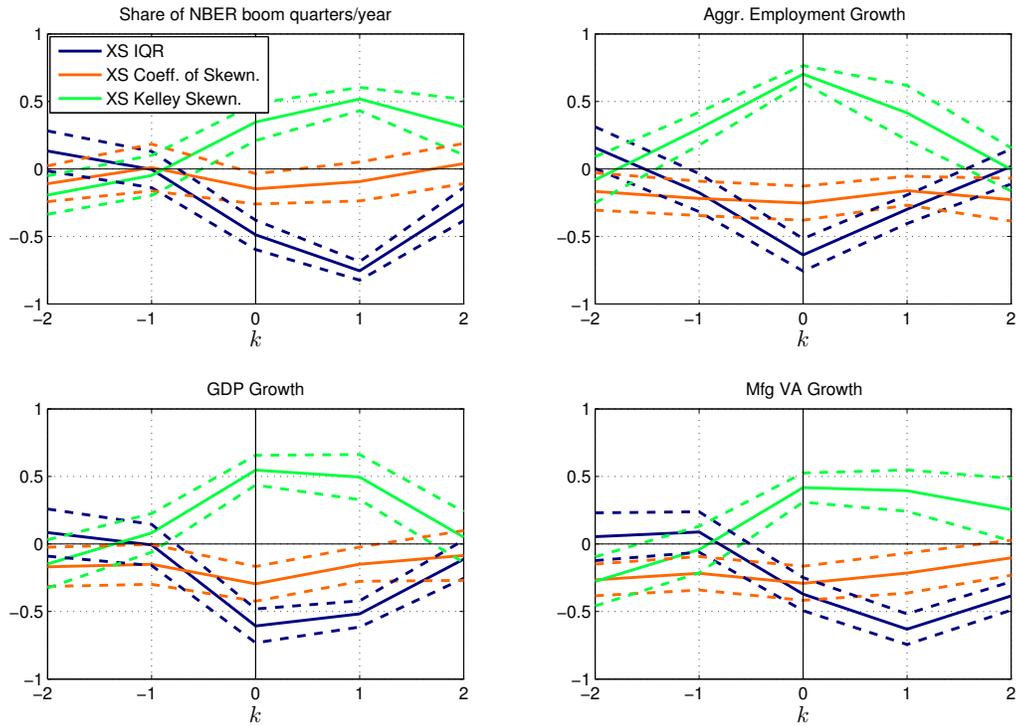
To check if the cyclical properties of employment growth outlined in Sections 3.2 and 3.3 are robust to the measure of the cycle, we plot several correlograms of second and third moments of both the cross-sectional and time-series employment growth distribution using various cyclical measures in Figures 14 and 15: the share of NBER boom quarters per year, the growth rates of aggregate employment, gross domestic product, manufacturing value added as well as the HP filtered residuals of the log-levels of the same variables. To keep the presentation succinct, we omit the the correlograms with HP filtered residuals.

As Figure 14 confirms the evidence displayed in Table 1: Across all measures of the cycle, employment growth dispersion across firms is distinctly countercyclical. This countercyclicity appears significant at the 5% level both contemporaneously and with a one-year lag. The correlation of cross-sectional skewness with the various cycle measures is less clear. While Kelley skewness is clearly procyclical at the 5% level contemporaneously and with a one-year lag, Pearson’s coefficient of skewness is slightly countercyclical, but the correlation coefficient is almost never significantly different from zero for any leads or lags and measure of the cycle. That the two skewness measures have opposing correlations with the cycle is still consistent with the predictions of Proposition 3. This proposition can only predict the cyclicity of skewness if the coefficient of absolute risk aversion of the estimated hiring rule (see Figure 8) does not monotonically increase or decrease with the profitability shock. We find that it is not.

Figure 15 plots the cyclicity of the “rolling-window” measures of time-series volatility and skewness. We have constructed such five-year rolling windows at the firm level, the industry level, the sectoral level and the level of the aggregate private economy. To keep the presentation concise, we only display the results of the two polar cases: the moments constructed at the firm level and at the level of the aggregate economy. Both rolling-window estimates of second and third moments have been detrended to account for the long-run decline in volatility (“Great Moderation”), though in case of the skewness there is almost no discernible trend. The second time-series moment at

Figure 14: Cyclicity of cross-firm dispersion and skewness

$$\text{Corr}(\text{Cycle}_t, \text{XS Empl. Moment}_{t+k})$$

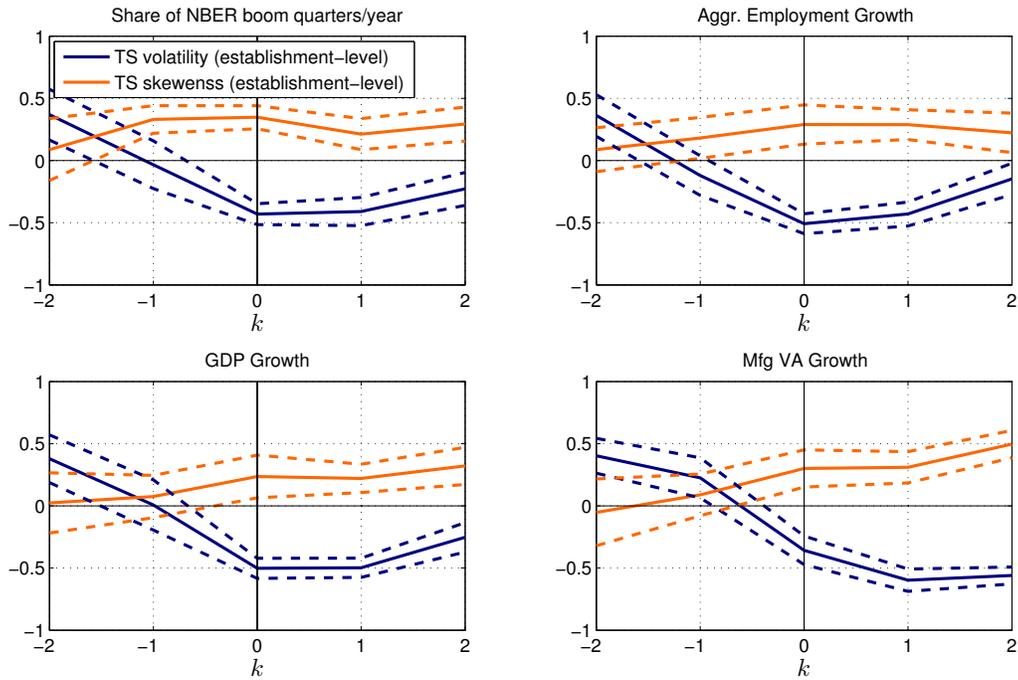


Note: Each plot displays the correlation of a particular measure of the cycle (indicated above the plot) with the cross-sectional employment growth inter-quartile range (blue), coefficient of skewness (orange) and Kelley skewness (green). Standard errors for the correlation coefficients are computed using a GMM procedure that corrects for heteroskedasticity and autocorrelation as in Newey and West (1987) and is adapted from Hansen et al. (1988).

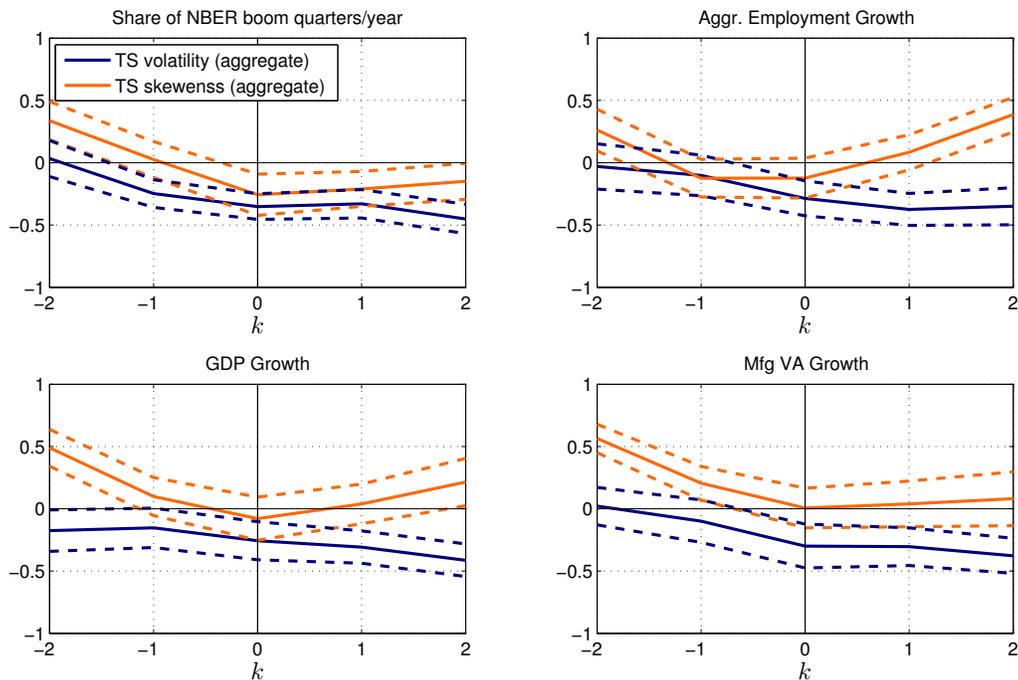
Figure 15: Cyclicity of time-series volatility and skewness of employment growth

$$\text{Corr}(\text{Cycle}_t, \text{TS Empl. Moment}_{t+k})$$

Panel A. Firm-level employment



Panel B. Aggregate employment (private economy)



Note: See notes to Figure 14.

all levels of aggregation are distinctly negatively correlated with the cycle. Average firm-level volatility is slightly more countercyclical than aggregate employment, but both are significantly countercyclical at the 5% level both contemporaneously and with a one-year lag. This means both firms, industries, sectors and the aggregate economy become more volatile in recessions.

Like in the cross section, time-series skewness does not exhibit an unambiguous cyclical pattern. At the aggregate level it is never significantly different from being acyclical. At the firm level, time-series skewness tends to be procyclical (like Kelley skewness in the cross-section), but its significance depends on the measure of the cycle: time-series skewness of the average firm is more positively correlated with the share of NBER boom quarters per year and aggregate employment growth as opposed to aggregate and manufacturing output growth measures where the positive correlation is barely significant. Interpreting the slight positive correlation, this means that firm firing is larger in absolute value than its hiring and this difference becomes more pronounced in recessions.

C.2 Alternative TFP measures

Our preferred measure for TFP shocks – the one used in producing Figures 8 and 9 as well as Tables 3, 4, 5 and 6 – is obtained by estimating the fixed-effects panel model in equation (11). The fixed effect causes a well-known bias in the estimated parameters and TFP innovations. Though Monte Carlo studies suggest the bias in a long panel like ours should be quite small (see Nickell (1981)), we re-estimate equation (11) using the instrumenting proposed by Arellano and Bond (1991) and Blundell and Bond (2000). The resulting measure of TFP innovations is not only extremely similar to the one we obtain in the fixed effects panel regression, the resulting employment growth-TFP innovation relationship is equally asymmetric. If the previously observed asymmetric hiring rule were a mere artifact of biased estimates of TFP innovations, then using the bias-corrected TFP innovations should yield a symmetric estimate of the hiring rule. This re-estimated hiring rule³⁰ is presented in Table 10 which shows that the hiring rule is still asymmetric and similar to the results presented in Table 6.

C.3 Properties of TFP shocks

This section presents second and third moments of TFP shocks underlying the same data cuts as in Table 5. The aim of this section is twofold: first, we want to demonstrate that employment patterns documented along these cuts are due to the strength of the concave TFP-employment growth relationship rather than linearly reflect properties of the underlying TFP shocks. Second, several micro-founded strands of the literature rest on assumptions that play out along a particular dimension we study in our data cuts. We want to help those researchers calibrate their models and study the strength of their mechanism along dimensions they are interested in. Table 11 presents the volatility and skewness of TFP shocks labeled z_t^i in the expression above equation (11).

C.4 Further analysis of composition effects

The asymmetric hiring rule we estimated previously could be an artifact of composition. We consider two of the most plausible ones: labor-saving capital investment and the firm's life cycle.

As for the first, the logic runs as follows: firms that receive a positive technology shock might invest in new more productive machinery that requires less labor input thus muting employment

³⁰For simplicity we focus on our preferred specifications (I) and (II) from Table 6, but the asymmetry continues to hold in other other specifications.

Table 10: Alternative Measures of TFP Innovations

$$\begin{aligned} \text{Regressions (I)/(III): } n_t^i &= \beta_0 + \beta^i + \beta_1 t + \beta_2 Z_{t-1}^i + \beta_3 l_t^i + \beta_4 z + \beta_5 z^2 \\ \text{Regressions (II)/(IV): } n_t^i &= \beta_0 + \beta^i + \beta_1 t + \beta_2 Z_{t-1}^i + \beta_3 l_t^i + \beta_4 z + \beta_7 z \mathbb{I}\{z < 0\} \end{aligned}$$

	Arellano-Bond		Blundell-Bond	
	(I)	(II)	(III)	(IV)
Explanatory variables	quadr.	piece-wise linear	quadr.	piece-wise linear
z	0.0715*** (0.0018)	0.0143*** (0.0029)	0.0746*** (0.0018)	0.0127*** (0.0029)
z^2	-0.0851*** (0.0035)		-0.0896*** (0.0046)	
$z \times \mathbb{I}\{z < 0\}$		0.1162*** (0.0045)		0.1186*** (0.0046)
Trend	-0.0032*** (0.0001)	-0.0032*** (0.0001)	-0.0032*** (0.0001)	-0.0032*** (0.0001)
Z_{t-1}^i	0.0252** (0.0015)	0.0251** (0.0015)	0.0273** (0.0014)	0.0273** (0.0014)
l_t^i	0.2972*** (0.0020)	0.2972*** (0.0020)	0.2963*** (0.0020)	0.2963*** (0.0020)
Constant	-1.3019*** (0.0389)	1.2961*** (0.0389)	1.2999*** (0.0389)	1.2941*** (0.0389)
Sample	ASM	ASM	ASM	ASM
N	1,536k	1,536k	1,536k	1,536k
R^2	0.1688	0.1688	0.1684	0.1684
Pos. Resp.	+1.42%	+0.39%	+1.34%	+0.33%
Neg. Resp.	-2.68%	-3.56%	-2.57%	-3.44%
Diff. sign.?	Yes***	Yes***	Yes***	Yes***

Note: *, **, *** significantly different from 0 at the 10%, 5%, 1% level, respectively.

growth after positive shocks. But after negative technology shocks firms might simply fire workers and divest machinery thus pronouncing an employment contraction after negative technology shocks. If that was true, then our observed asymmetry would be caused by firms that upgrade their technology. We therefore re-estimate our preferred linear-quadratic specification only for firms that do not simultaneously invest large amounts (which we identify as possibly labor-saving). Following the literature, we define a large investment project as exceeding an investment rate of 0.20. Column (I) in Table 12, however, confirms that the asymmetry is still present, significant and quantitatively close to what we estimated before, even for firms that do not upgrade their capital stock by large amounts. Second, we want to check if the observed asymmetry is driven by life-cycle considerations. To that end, we focus on “mid-age” firms, which are less subject to dynamics specific to entry or exit. We define these as firms three years or more after their creation and three years or more before their death.³¹ Column (II) in Table 12 shows that the asymmetry does not disappear when

³¹We identify death in the data as exit in the Longitudinal Business Database rather than disappearance from our

Table 11: Dispersion and skewness of TFP shocks along various dimensions

Bin	Parent firm characteristics			Firm characteristics			Industry characteristics		
	(I) Size (empl.)	(II) No. units/ parent firm	(III) Public/ Private	(IV) Size (empl.)	(V) Share NP empl.	(VI) Age	(VII) Capital Intensity	(VIII) Industry volatility	(IX) Durables/ non-durables
<i>A. Cross-sectional dispersion</i>									
1	0.176	0.163	0.175	0.193	0.205	0.183	0.205	0.154	0.169
2	0.165	0.171	0.180	0.182	0.174	0.174	0.175	0.165	0.171
3	0.161	0.178		0.177	0.163	0.168	0.167	0.172	
4	0.166	0.180		0.174	0.160	0.167	0.165	0.154	
5	0.177	0.180		0.169	0.161	0.174	0.164	0.177	
6	0.181	0.179		0.169	0.169	0.176	0.167	0.166	
7	0.182	0.183		0.172	0.181	0.188	0.174	0.223	
8	0.184	0.186		0.181	0.211	0.206	0.201	0.248	
<i>B. Cross-sectional skewness</i>									
1	0.068	0.098	0.113	0.035	0.257	0.061	0.300	0.035	0.024
2	0.077	0.080	-0.007	0.090	0.103	0.051	0.151	0.021	0.093
3	0.097	0.082		0.092	0.066	0.012	0.133	0.073	
4	0.102	0.062		0.074	0.046	-0.007	0.077	0.017	
5	0.075	0.066		0.048	0.027	0.037	0.029	0.051	
6	0.085	0.023		0.069	0.038	0.137	-0.007	0.087	
7	0.040	0.060		0.083	0.026	0.253	0.008	0.119	
8	0.055	0.070		0.104	0.047	0.333	-0.042	0.151	

Note: For descriptions of bins see Table 5.

focusing on mid-age firms.

Table 12: Is Asymmetry Driven by Labor-Saving Capital Investment? By Life-Cycle Patterns?

$$\text{Regressions (I)-(IV): } n_t^i = \beta_0 + \beta^i + \beta_1 t + \beta_2 Z_{t-1}^i + \beta_3 l_t^i + \beta_4 z + \beta_5 z^2$$

Explanatory variables	(I)	(II)	(III)	(IV)
	no large investments	≥ 3 years	≥ 5 years	≥ 8 years
		away from both firm birth and death		
β_0	-1.3628*** (0.0391)	-1.4048*** (0.0387)	-1.4042*** (0.0322)	-1.4279*** (0.0372)
β_1	-0.0025*** (0.0001)	-0.0022*** (0.0001)	-0.0014*** (0.0001)	-0.0009*** (0.0001)
β_2	0.0247*** (0.0015)	0.0311*** (0.0014)	0.0313*** (0.0015)	0.0301*** (0.0013)
β_3	0.3010*** (0.0020)	0.2983*** (0.0020)	0.2986*** (0.0021)	0.2981*** (0.0019)
β_4	0.0750*** (0.0019)	0.0759*** (0.0018)	0.0724*** (0.0021)	0.0707*** (0.0024)
β_5	-0.1415*** (0.0064)	-0.1459*** (0.006)	-0.1575*** (0.0061)	-0.1600*** (0.0059)
Sample	ASM $\frac{i}{k} < 0.2$	ASM mid-age 1	ASM mid-age 2	ASM mid-age 3
N	1,406k	1,421k	1,262k	1,132k
R^2	0.178	0.190	0.188	0.191
Pos. Response	+0.89%	+0.89%	0.79%	0.75%
Neg. Response	-1.79%	-1.82%	-1.80%	-1.77%
Diff. sign.?	Yes***	Yes***	Yes***	Yes***

Note: *, **, *** significantly different from 0 at the 10%, 5%, 1% level, respectively.

We thus conclude that compositional effects driven by a firm's capital goods upgrade and a firm's life-cycle growth and decline are not responsible for the observed asymmetric hiring rule.

C.5 Alternative specifications for quantitative significance

In this appendix we report results based on alternative specifications to the benchmark signal structure and parametric function of in equation (14). In Table 13 we collect the moments defined in section 5.3. The first two columns give their informal and formal description. Column (M1) reports values for the benchmark specification. The rest of the remaining columns are based on alternative model specifications to be introduced below.

One extension is to allow for noise in the idiosyncratic signal. More specifically, as in equation (6), we write the general representation of the signal as

$$s_t^i = u_t^a + v_t^a + u_t^i + v_t^i$$

main manufacturing sample.

where v_t^a and v_t^i are the common and idiosyncratic components of the noise distributed as $v_t^a \sim \mathcal{N}(0, \sigma_{v,a}^2)$ and $v_t^i \sim \mathcal{N}(0, \sigma_{v,b}^2)$, respectively. We parametrize each of the variances of the noise through the implied signal to noise ratios

$$\kappa_x = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_{v,x}^2}$$

for both the aggregate and idiosyncratic components, i.e. $x = a, b$. Allowing for noise in the signals means that the econometrician recovers the conditional expectation $g(z_t^i) = E[f(s_t^i)|z_t^i]$, as defined in equation (7). We discuss some of the conclusions that can be drawn about $f(s_t^i)$ based on the observed $g(z_t^i)$ in Section 2.3. In this context however, we maintain for expositional purposes the assumption that $f(s_t^i)$ is still given by the linear-quadratic function in (14), with the same coefficients β^1 and β^2 as those estimated for the estimated $g(z_t^i) = \beta^1 z_t^i + \beta^2 (z_t^i)^2$. Thus, in the experiments reported below, allowing for noise amounts to maintaining the same decision rule while increasing the variance of either the idiosyncratic component, $u_t^i + v_t^i$, or the aggregate one, given by $u_t^a + v_t^a$.

In column (M2), we set $\kappa^a = \kappa^b = 0.5$, so that the variance $\sigma_{v,a}^2 = \sigma_a^2$ and $\sigma_{v,b}^2 = \sigma_i^2$. We observe that all statistics increase as we have essentially made both types of shocks more volatile. The larger volatility will have larger effects through the same curvature of the hiring rule. In column (M3), we keep $\kappa^a = 1$, so that there is no aggregate noise, but activate $\kappa^b = 0.5$. We see that the properties of cross-sectional and time series skewness are the same as in column (M2) since the amount of idiosyncratic variance is the same. The cyclical nature of the cross-sectional dispersion is very similar to the benchmark case, with a very small difference caused by the larger variance of idiosyncratic noise. Lastly, in column (M4) we revert to $\kappa^b = 1$ and keep $\kappa^a = 0.5$. Similarly to the above logic, the cross-sectional and time series skewness are the same as the benchmark case while the cyclical nature of the cross-sectional dispersion is larger.

A final alternative specification that we investigate is to change the hiring decision rule to a piece-wise linear function in which employment growth continues to respond stronger to bad than to good signals. We maintain the benchmark assumption of no additional noise in the signal and consider the decision rule

$$f(s_t^i) = \beta_4 s_t^i + \beta_7 s_t^i \mathbb{I}\{s_t^i < 0\}$$

where the coefficients β_4 and β_7 equal 0.0347 and 0.0947, respectively, as estimated for specification (III) in Table 6. Column (M5) reports the results for this case. We see that the main qualitative features are maintained and that the piece-wise linear function in general produces stronger quantitative effects.

Table 13: Quantitative significance of estimated concavity of hiring decision rule

Distribution Moment		(M1)	(M2)	(M3)	(M4)	(M5)
Description	Statistic					
XS dispersion cyclicalilty	$\ln \left[\frac{IQR(n_t^i u_t^a = -\sigma_a)}{IQR(n_t^i u_t^a = \sigma_a)} \right]$	22%	32%	23%	31%	36%
XS skewness	$\gamma(n_t^i u_t^a = 0)$	-1.17	-1.55	-1.55	-1.17	-1.01
TS skewness	$\frac{1}{N} \sum_{i=1}^N \frac{1}{T} \sum_{t=1}^T \frac{(n_t^i - \bar{n}^i)^3}{(Vol^i)^{3/2}}$	-1.12	-1.49	-1.49	-1.12	-0.99

Note: Column 1 and 2 give the informal and formal description of the distribution moments, where XS refers to cross-section and TS to time series. Column (M1) the corresponding values for the benchmark specification. Column (M2) refers to the alternative specification with noise, where $\kappa^a = \kappa^b = 0.5$; (M3) with $\kappa^a = 1, \kappa^b = 0.5$ (M4) with $\kappa^a = 0.5, \kappa^b = 1$ and (M5) is the piece-wise linear model without noise.