LINEAR MODELS – POPULATION GROWTH IN FIVE STATES

TEACHER VERSION

Subject Level:
Middle School Math

Grade Level:
8

Approx. Time Required:
45-60 minutes

Learning Objectives:

- Students will be able to examine a linear model to estimate the slope and *y*-intercept.
- Students will be able to interpret the slope and *y*-intercept.
- Students will be able to understand how bivariate data can be simplified to make a linear model.

United States™ Census Bureau

Classrooms Powered by Census Data

STATISTICS IN SCHOOLS

Activity Description

Students will look at decennial census data — in table and graph form — showing population growth trends in five states from 1950 to 2010. Students will also estimate and interpret lines of best fit.

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Topics:
- Bivariate data
- Linear equations
- Linear models
- Population
- Rounding
- Scatter plots
- Slope
- *y*-intercept

Skills Taught:
- Calculating slope
- Calculating the *y*-intercept
- Interpreting slope and *y*-intercept
- Working with bivariate data

Materials Required

- The student version of this activity, 16 pages; it contains images that should be printed in color.
- Rulers

Calculators are optional.

Activity Items

The following items are part of this activity. Items, their data sources, and any relevant instructions for viewing the source data online appear at the end of this teacher version.

- Item 1: Five States' Growing Populations by the Numbers: 1950–2010
- Item 2: Five States' Growing Populations in a Graph: 1950–2010

For more information to help you introduce your students to the Census Bureau, read "*Census Bureau 101 for Students*." This information sheet can be printed and passed out to your students as well.

Standards Addressed

See charts below. For more information, read "*Overview of Education Standards and Guidelines Addressed in Statistics in Schools Activities*."

Common Core State Standards for Mathematics

Standard	Domain	Cluster
CCSS.MATH.CONTENT.8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	8 SP – Statistics & Probability	Investigate patterns of association in bivariate data.
CCSS.MATH.CONTENT.8.SP.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.	8 SP – Statistics & Probability	Investigate patterns of association in bivariate data.

Common Core State Standards for Mathematical Practice

Standard
CCSS.MATH.PRACTICE.MP4. Model with mathematics. Students will model population growth in five states using scatter plots and lines of best fit.
CCSS.MATH.PRACTICE.MP6. Attend to precision. Students will interpret the meaning of slope and the y-intercept in the context of population growth.

National Council of Teachers of Mathematics' Principles and Standards for School Mathematics

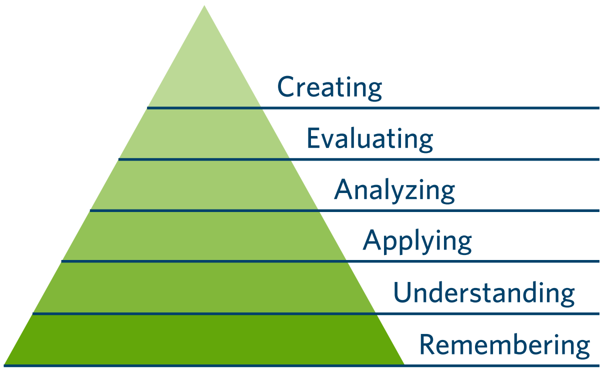
Content Standard	Students should be able to:	Expectation for Grade Band
Algebra	Analyze change in various contexts.	Use graphs to analyze the nature of changes in quantities in linear relationships.

Guidelines for Assessment and Instruction in Statistics Education

GAISE	Level A	Level B	Level C
Formulate Questions	X		
Collect Data			
Analyze Data		X	
Interpret Results		X	

Bloom's Taxonomy

Students will ***apply*** their understanding of scatter plots and linear models to ***analyze*** the features of lines of best fit.



Teacher Notes

Before the Activity

Students must understand the following key terms:

- **Bivariate data** – pairs of linked numerical observations (e.g., a list of heights and weights for each player on a football team)
- **Scatter plot** – a graph in the coordinate plane that displays a set of bivariate data and can be used to determine how two variables are associated (e.g., to show associations between the heights and weights of players on a football team)
- **Line of best fit** – a straight line drawn through the center of a group of data points on a scatter plot, showing how closely the two variables on the scatter plot are associated
- **Linear model** – a line of best fit that is used to predict *y* values based on x values
- **Slope** – the rate of change in a linear model or the strength and direction of association between variables in a linear regression
- ***y*-intercept** – the point where a line crosses the *y*-axis of a graph and where the *x* value is zero

Students should have a basic understanding of the following concepts:

- Graphing ordered pairs (*x, y*) to make scatter plots
- Recognizing linear relationships (especially those written in slope-intercept form)

Teachers could have students complete the optional introduction (an estimated additional 15–20 minutes) to get them familiar with estimating and interpreting slope and the *y*-intercept, if they are not already.

During the Activity

Teachers could have students use calculators to help them find the slopes of the linear models throughout the activity.

After students complete part 2, teachers should pause to tell them that making predictions off the grid (extrapolation) risks accuracy and should be viewed with skepticism.

Teachers could also have students work in groups of four to complete the first four questions in part 3, with each student answering one of the questions; then teachers could direct students who answered the same question to compare their answers.

After the Activity

Teachers should ask students the following questions to spark a class discussion:

- Why do we sometimes round data before graphing them, and what are the disadvantages of doing so? (Teachers should expect varied student answers that mention how rounding simplifies a graph while also compromising some accuracy.)
- Why is looking at data in context so important? (Teachers should expect answers about how data tell a story, the context of those data helps readers understand that story, and interpretations of data can vary depending on context.)
- Do all bivariate data follow a linear model? (The correct answer is "no.")
- How is a linear model different from a linear equation? (Teachers may need to help students understand that a linear model has context, with its x and y values in this case representing years and numbers of people, and it is used to predict y values. While actual data points won't necessarily fall on the line of best fit of a linear model, a linear equation perfectly represents the data points and so is not predictive.)

Extension Ideas

- Teachers could provide students with an interpretation of a linear model and ask them to work backward to find the linear equation and/or create a scatter plot. For example: "According to the 2007 Census, 12.6 percent of our state's population was older than 70, and the Census Bureau predicted that this percentage would increase by 0.6 percentage points per year."
- Teachers could have students find population data over time for a state not included in this activity, create a scatter plot of those data, and estimate a line of best fit.
- Teachers could ask students to find 1950–2010 population data for a different state and compare its growth (or shrinkage) with the population growth of the states in this activity. Then students could research why the state population grew (or shrunk).
- Teachers could have students compare the population growth in a state with the overall growth in U.S. population over time (which teachers could provide or have students find through research).

Student Activity

Click *here* to download a printable version for students.

Activity Items

The following items are part of this activity and appear at the end of this student version.

- Item 1: Five States' Growing Populations by the Numbers: 1950–2010
- Item 2: Five States' Growing Populations in a Graph: 1950–2010

Student Learning Objectives

- I will be able to examine a linear model to estimate the slope and *y*-intercept.
- I will be able to interpret the slope and *y*-intercept.
- I will be able to understand how bivariate data can be simplified to make a linear model.

Introduction–Estimate and Interpret the Slope and y-Intercept (Optional)

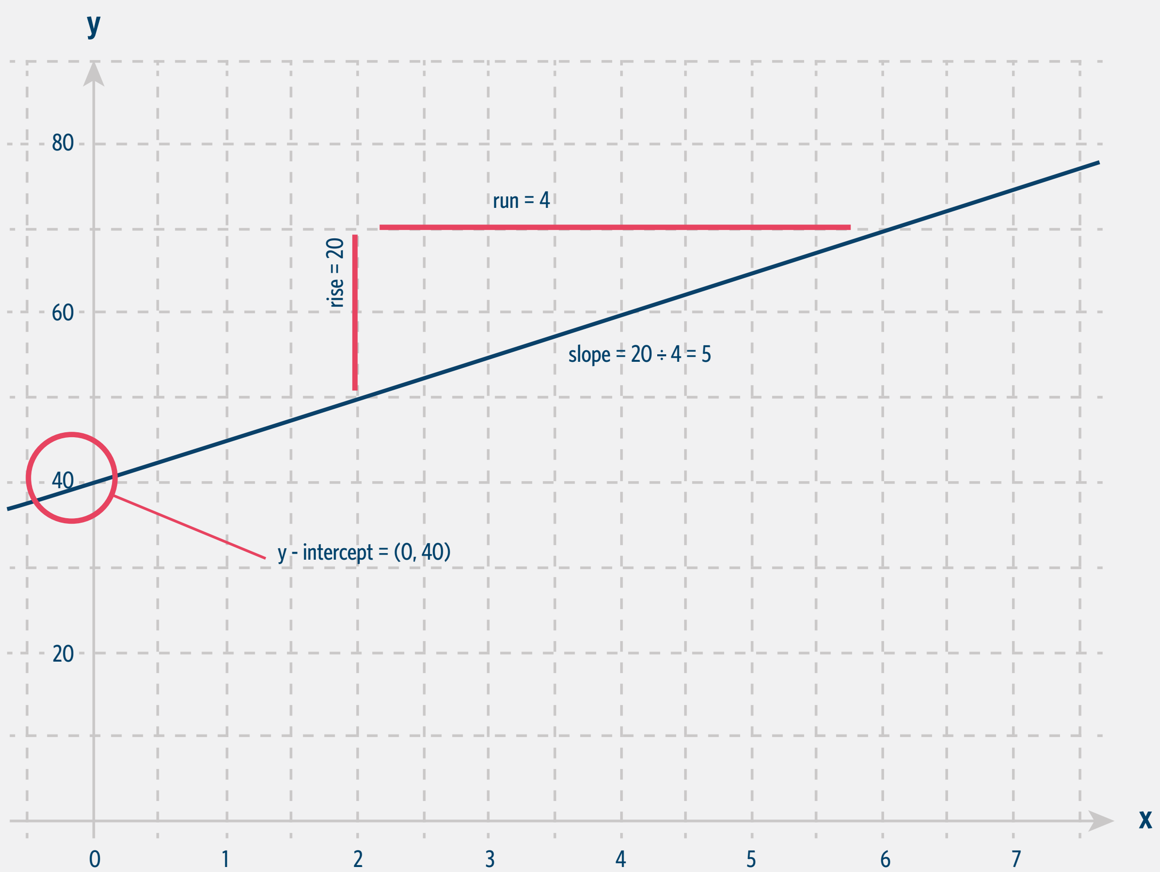
Read the example and then fill in the rest of this table:

	Slope		*y*-Intercept	
	What is the slope?	What does the slope mean in the context of these data?	What are the coordinates of the *y*-intercept?	What does the *y*-intercept mean in the context of these data, and does that finding make sense?
$y = 5x + 200$, EXAMPLE: where *y* is the cost of making T-shirts (in dollars) and x is the number of T-shirts made	5	On average, for each additional T-shirt made, the cost increases by $5.	(0, 200)	It costs $200 to make no T-shirts. This makes sense! (It costs money upfront to set up a T-shirt factory before the factory can start making shirts.)

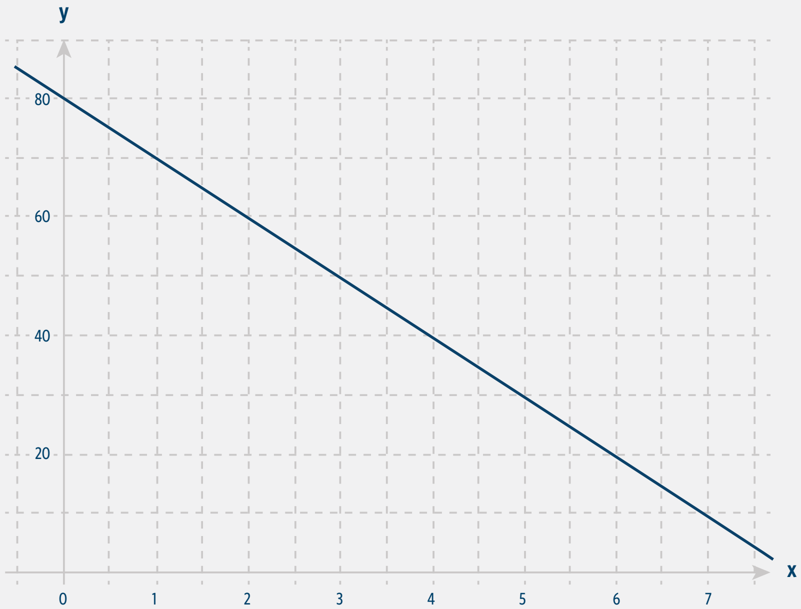
	Slope		y-Intercept	
$y = -3x + 40$, where y is the number of turtles in a display and x is the number of weeks the turtles have been in the display	-3	On average, for each additional week that passes, the number of turtles in the display decreases by 3.	(0, 40)	There are 40 turtles in the display when it opens. This makes sense!
$y = 4x + 20$, where y is the height of a toddler in inches and x is the age of the toddler in years	4	On average, for each additional year of a toddler's life, the toddler's height increases by 4 inches.	(0, 20)	A child could be 20 inches long at birth. This makes sense!
$y = -0.4x + 3.20$, where y is the price of a notebook in dollars and x is the number of weeks the notebook has been on sale	-0.4	On average, for each additional week the notebook is on sale, the price goes down $0.40.	(0, 3.20)	The notebook costs $3.20 before it goes on sale. This makes sense!
$y = 0.25x + 0.5$, where y is the amount of homework assigned (in hours) and x is the grade level in school (1 for first grade, 2 for second grade, etc.)	0.25	On average, for each additional grade level in school, the amount of homework increases by a quarter of an hour.	(0, 0.5)	A child could have a half-hour of homework before first grade, like in kindergarten. This makes sense!

Determine the approximate slope and *y*-intercept of the lines on each of the following graphs by choosing any two points along the line, using this example to help you:

EXAMPLE:



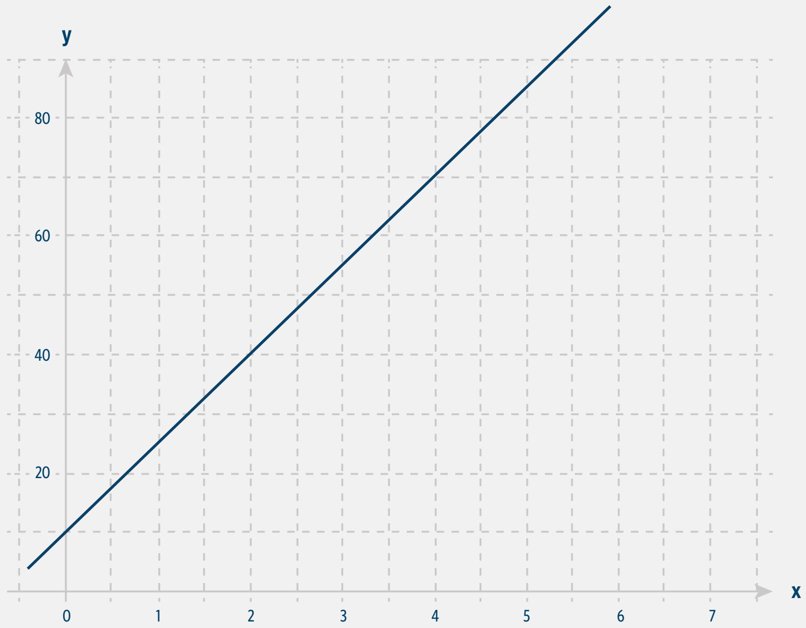
1.



Slope = **Student answers will vary depending on their approximate points chosen from the line. Using (3, 50) and (5, 30), the slope is -20 ÷ 2 = -10.**

y-intercept = **(0, 80)**

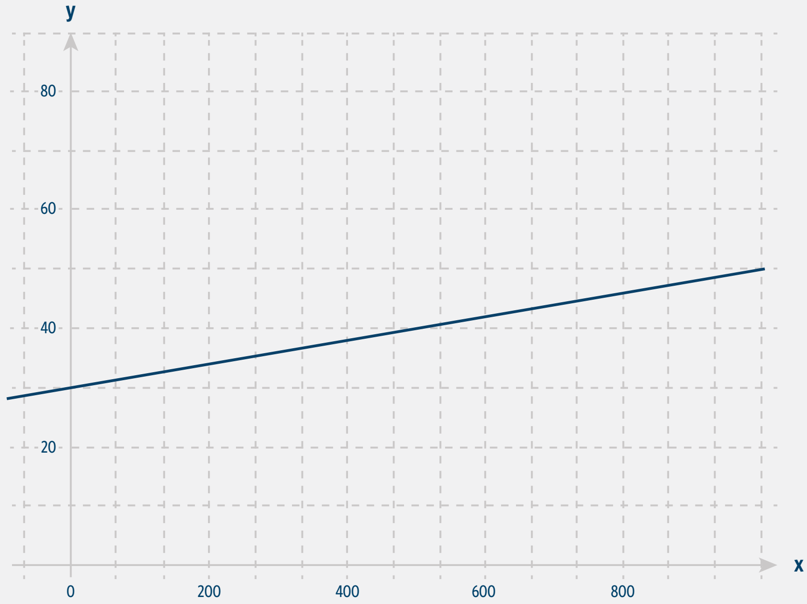
2.



Slope = **Student answers will vary depending on their approximate points chosen from the line. Using (2, 40) and (4, 70), the slope is 30 ÷ 2 = 15.**

y-intercept = **(0, 10)**

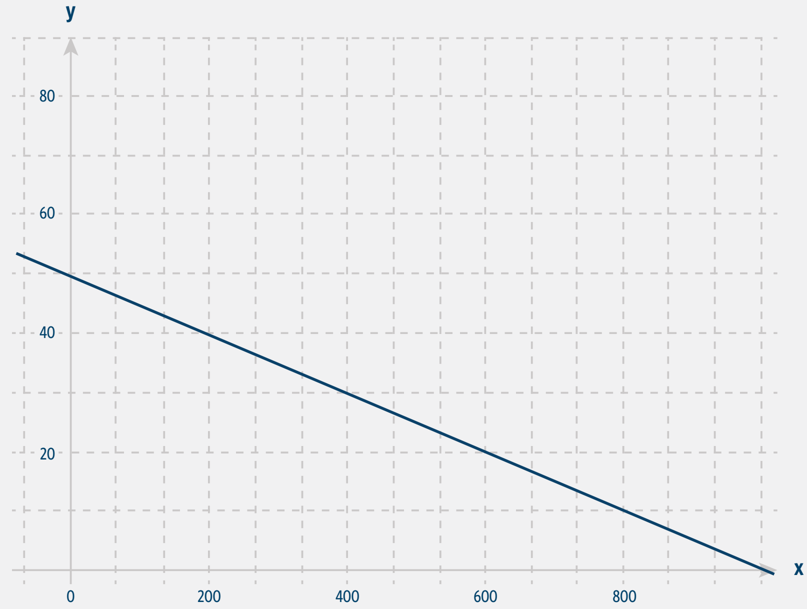
3.



Slope = **Student answers will vary depending on their approximate points chosen from the line. Using (0, 30) and (500, 40), the slope is 10 ÷ 500 = 0.02.**

y-intercept = **(0, 30)**

4.



Slope = **Student answers will vary depending on their approximate points chosen from the line. Using (400, 30) and (800, 10), the slope is -20 ÷ 400 = -0.05.**

y-intercept = **(0, 50)**

Part 1 - Simplify Data

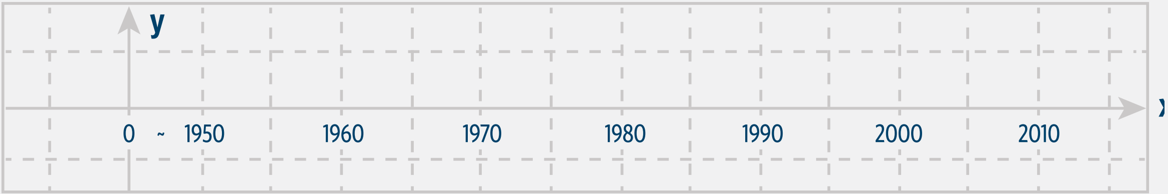
Use **Item 1: Five States' Growing Populations by the Numbers: 1950–2010** to answer the following questions.

1. How were the years changed from the first table to look simpler in the second table?

   The year 1950 was simplified to be the starting point of the table, at 0. Each decade after that is represented by a multiple of 10.

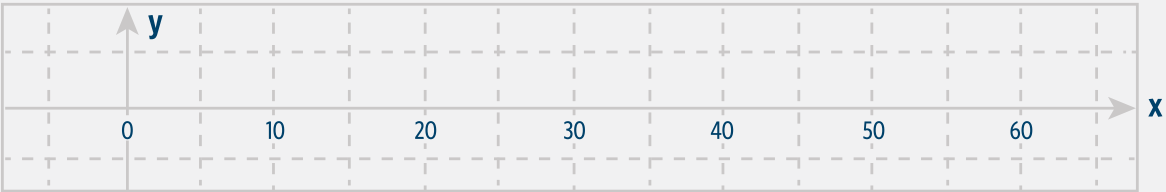
2. The following are a few ways to represent the years for these data on the horizontal axis of a graph. How would you label each axis?

A



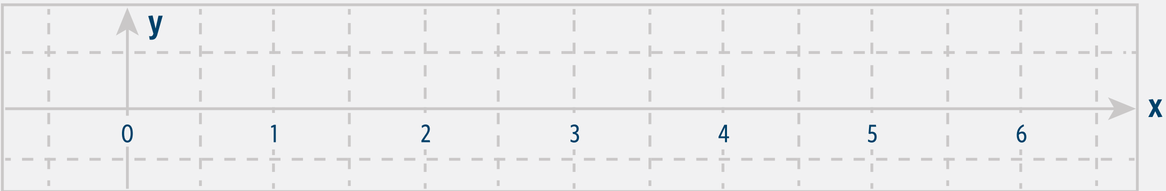
Horizontal Axis Label: **Year**

B



Horizontal Axis Label: **Years Since 1950**

C



Horizontal Axis Label: **Decades Since 1950**

Part 2 – Interpret Visual Data

Use **Item 2: Five States' Growing Populations in a Graph: 1950–2010** to answer the following questions.

1. Which state had the fastest-growing population between 1950 and 2010, and why do you think it grew so rapidly?

   California. Student explanations for that growth will vary but could include climate preferences, job opportunities, or the attraction of living near celebrities in Los Angeles and other areas of California.

2. Which state had a population of about 3 million in 1950?

   Georgia

3. Of the five states, how many had more than approximately 20 million people in 2010, and which states were they?

   Two: Texas and California

4. Between 1950 and 2010, when did California's population first exceed approximately 20 million?

   1970

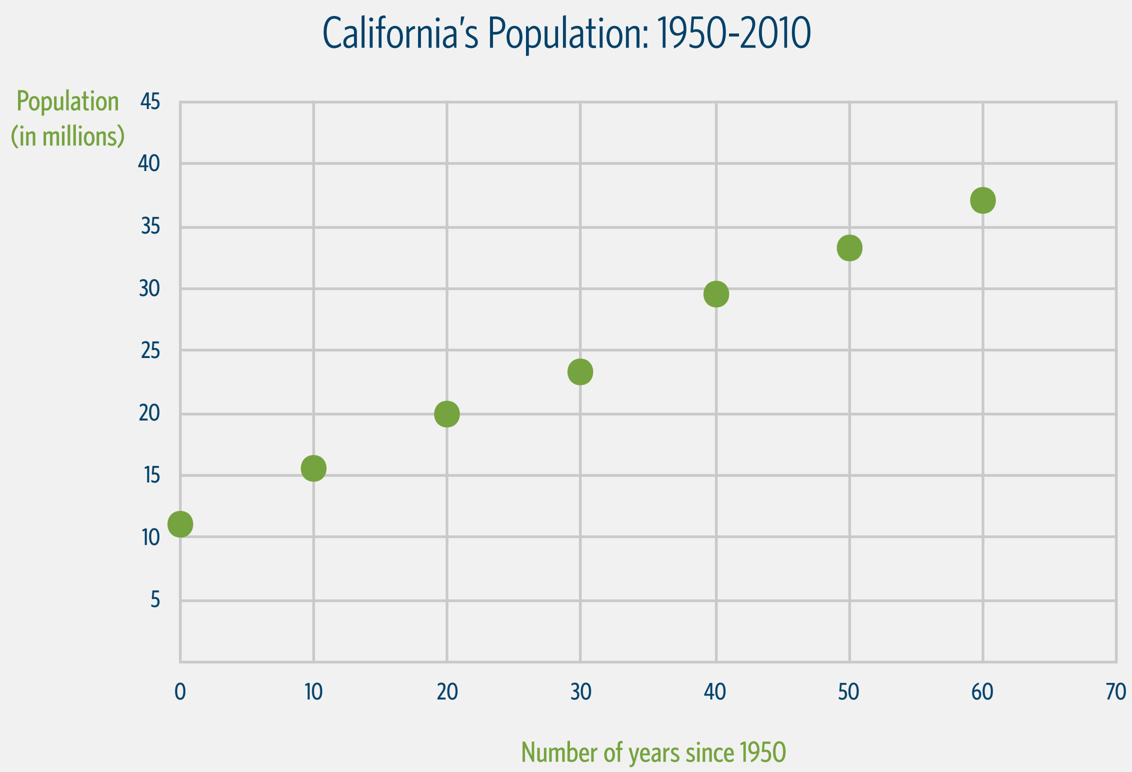
5. Looking at the graph, predict when Georgia will have a population of about 20 million.

   Student answers will vary but should be between 2040 and 2060.

Part 3 – Work With Linear Models

For questions 1–4 below, use **Item 1: Five States' Growing Populations by the Numbers: 1950–2010** to plot the states' populations on the graph templates provided.

1. The linear equation for the following graph is: $y = 0.45x + 11$, meaning the predicted population for California = 0.45(year) + 11.

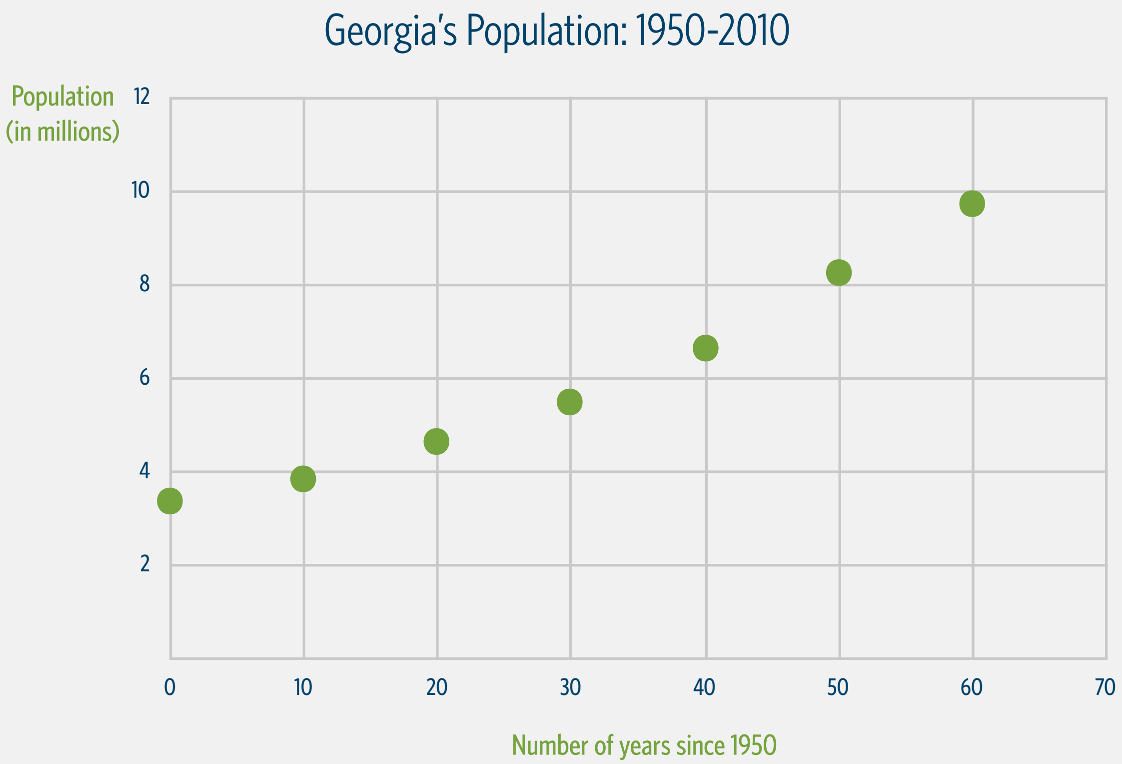


a. Write the coordinates of the *y*-intercept here: **(0, 11)**

b. Write the slope here: **0.45**

c. According to the estimated line of best fit, in 1950 (when *x* equals 0), California's population was approximately **11** million, and it has grown by an average of **0.45** million per year.

2. The linear equation for the following graph is: $y = 0.1x + 2.8$, meaning the predicted population for Georgia = 0.1(year) + 2.8.

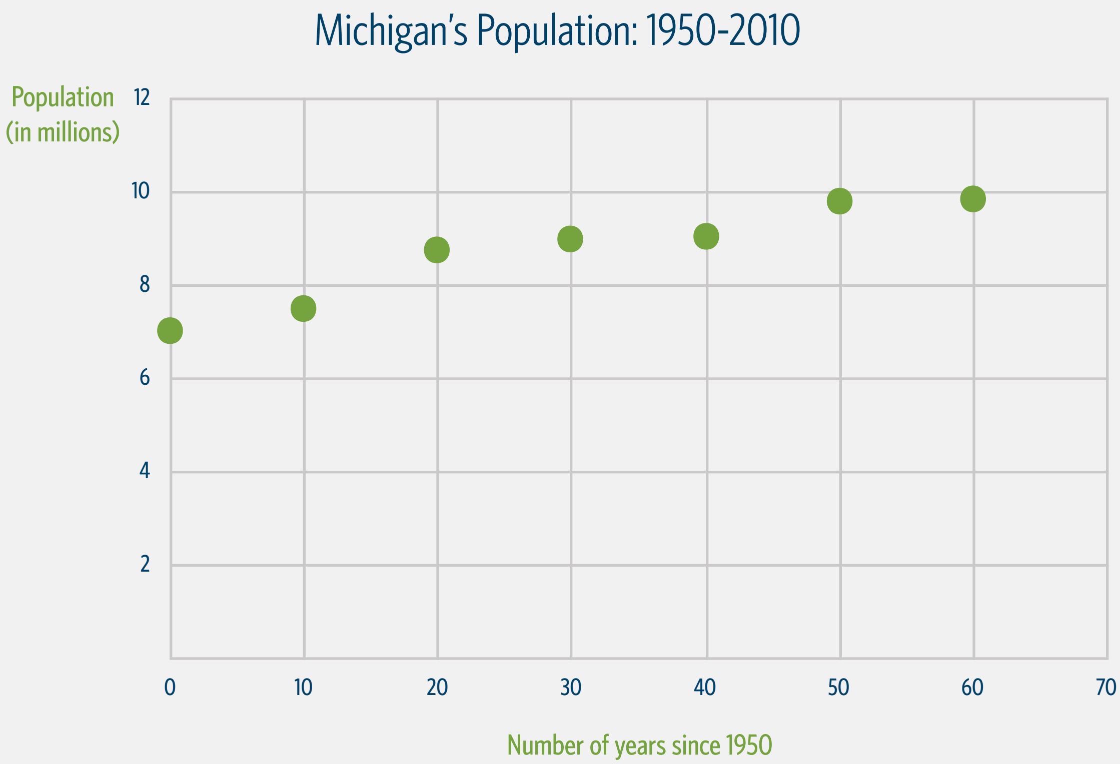


a. Write the coordinates of the y-intercept here: **(0, 2.8)**

b. Write the slope here: **0.1**

c. According to the estimated line of best fit, in 1950 (when x equals 0), Georgia's population was approximately **2.8** million, and it has grown by an average of **0.1** million per year.

3. The linear equation for the following graph is: $y = 0.05x + 7$, meaning the predicted population for Michigan = 0.05(year) + 7.

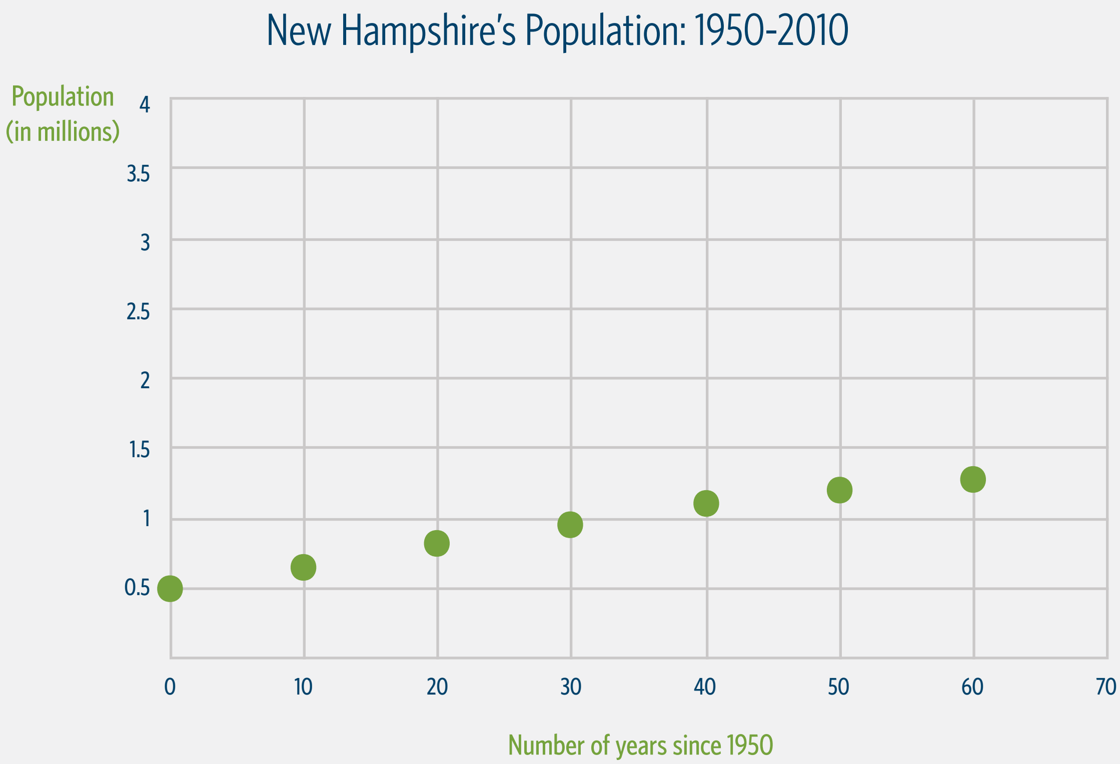


a. Write the coordinates of the y-intercept here: **(0, 7)**

b. Write the slope here: **0.05**

c. According to the estimated line of best fit, in 1950 (when x equals 0), Michigan's population was approximately **7** million, and it has grown by an average of **0.05** million per year.

4. The linear equation for the following graph is: $y = 0.014x + 0.5$, meaning the predicted population for New Hampshire = 0.014(year) + 0.5.

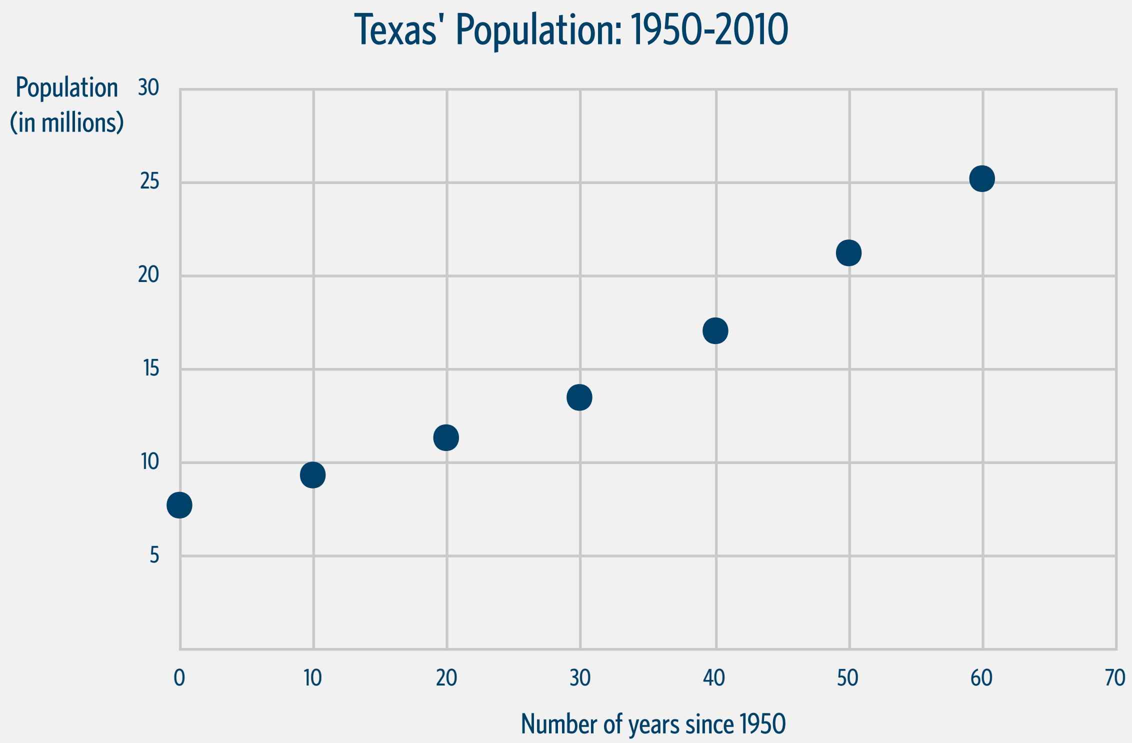


a. Write the coordinates of the y-intercept here: **(0, 0.5)**

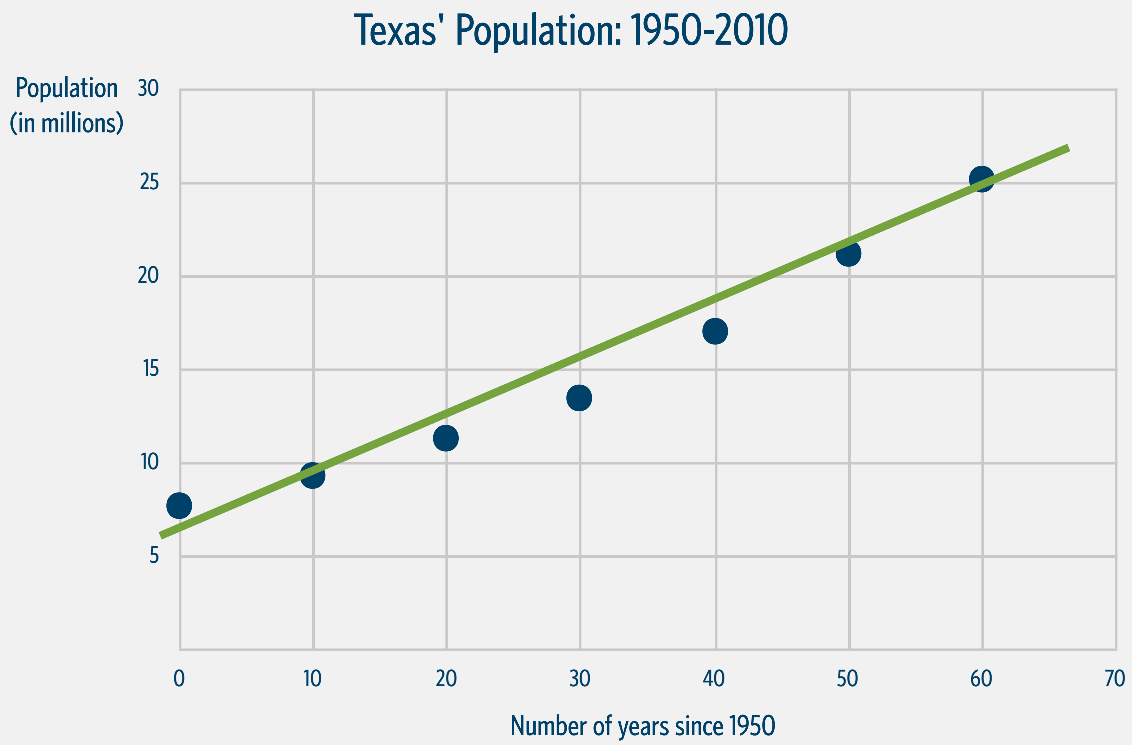
b. Write the slope here: **0.014**

c. According to the estimated line of best fit, in 1950 (when x equals 0), New Hampshire's population was approximately **0.5** million, and it has grown by an average of **0.014** million per year.

5. Using a ruler, draw a line of best fit on the following graph.



Student lines of best fit should look similar to:



a. Approximately where does your line intersect the vertical axis?

Student answers will vary but should be at about (0, 6).

b. Interpret your estimated *y*-intercept in the context of the data.

In 1950 (when x equals 0), there were about 6 million people living in Texas.

c. Using rise ÷ run, what is the estimated slope of your line?

Student answers will vary but should be around 0.3 to 0.4.

d. Interpret this slope in the context of the data.

On average, the population of Texas since 1950 has been growing at a rate of 0.3 million (or 300,000) people per year.

Part 4 – Apply What You Learned About Linear Models

Refer to the state graphs you just reviewed to answer the following questions in context.

1. What does "*x*" stand for?

   Number of years since 1950

2. What does "*y*" stand for?

   Population in millions

3. If the *y*-intercept for a state graph were 7, what would that mean?

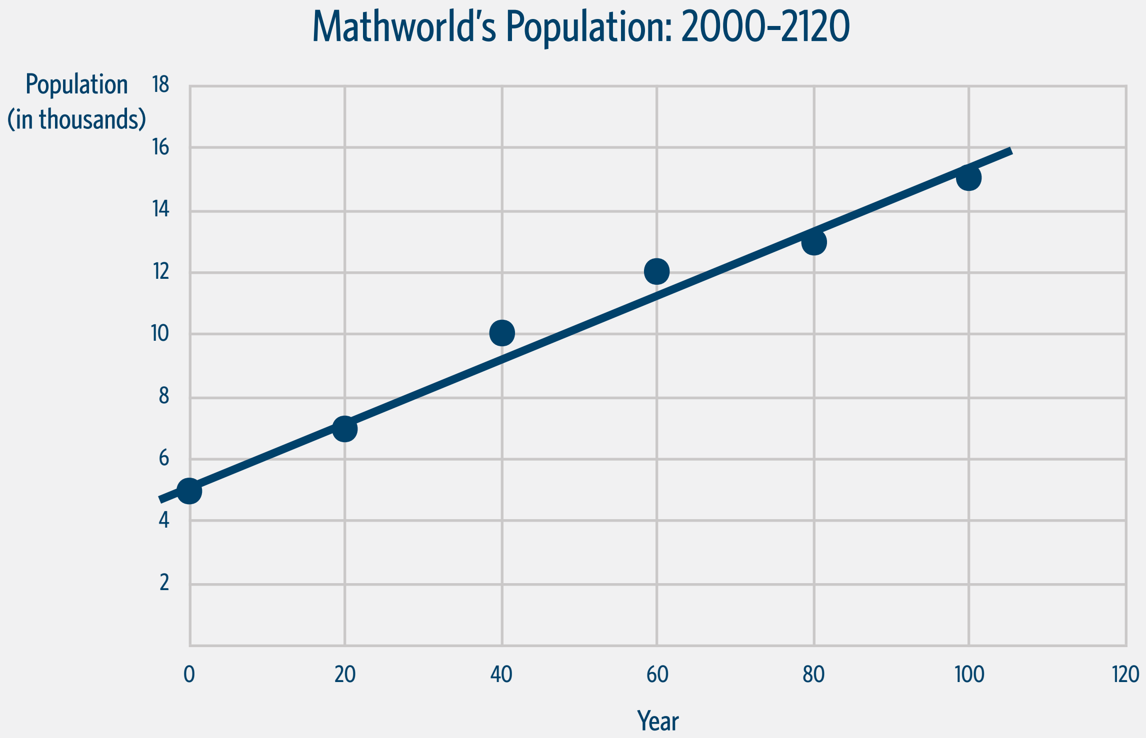
   The population of that state was 7 million in 1950.

4. If the slope of a line of best fit on a state graph were 0.6, what would that mean?

   The population of that state grew an average of 0.6 million (600,000 people) per year.

Use the following graph, showing the population growth of a mythical village called Mathworld, to complete the prompts that follow.

5. The equation of the line of best fit is $y = 0.1x + 5$. Determine the y-intercept and slope, and interpret them in the context of the data.



Coordinates of the y-intercept: **(0, 5)**

Slope: **0.1**

Interpretation: **In 2000, the population of Mathworld was about 5 million. Mathworld's population is predicted to grow by an average of 0.1 thousand (100 people) per year.**

6. Imagine that you have just presented this graph to an audience in a report about Mathworld. Write three questions your audience could ask you, and provide the answer for each in the table below.

Student questions and answers below will vary.

Question	Answer
1.	1.
2.	2.
3.	3.

Item 1: Five States' Growing Populations by the Numbers, 1950-2010

	1950	1960	1970	1980	1990	2000	2010
California	10,586,223	15,717,204	19,953,134	23,667,902	29,760,021	33,871,648	37,253,956
Georgia	3,444,578	3,943,116	4,589,575	5,463,105	6,478,216	8,186,453	9,687,653
Michigan	6,371,766	7,823,194	8,875,083	9,262,078	9,295,297	9,938,444	9,883,640
New Hampshire	533,242	606,921	737,681	920,610	1,109,252	1,235,786	1,316,470
Texas	7,711,194	9,579,677	11,196,730	14,229,191	16,986,510	20,851,820	25,145,561

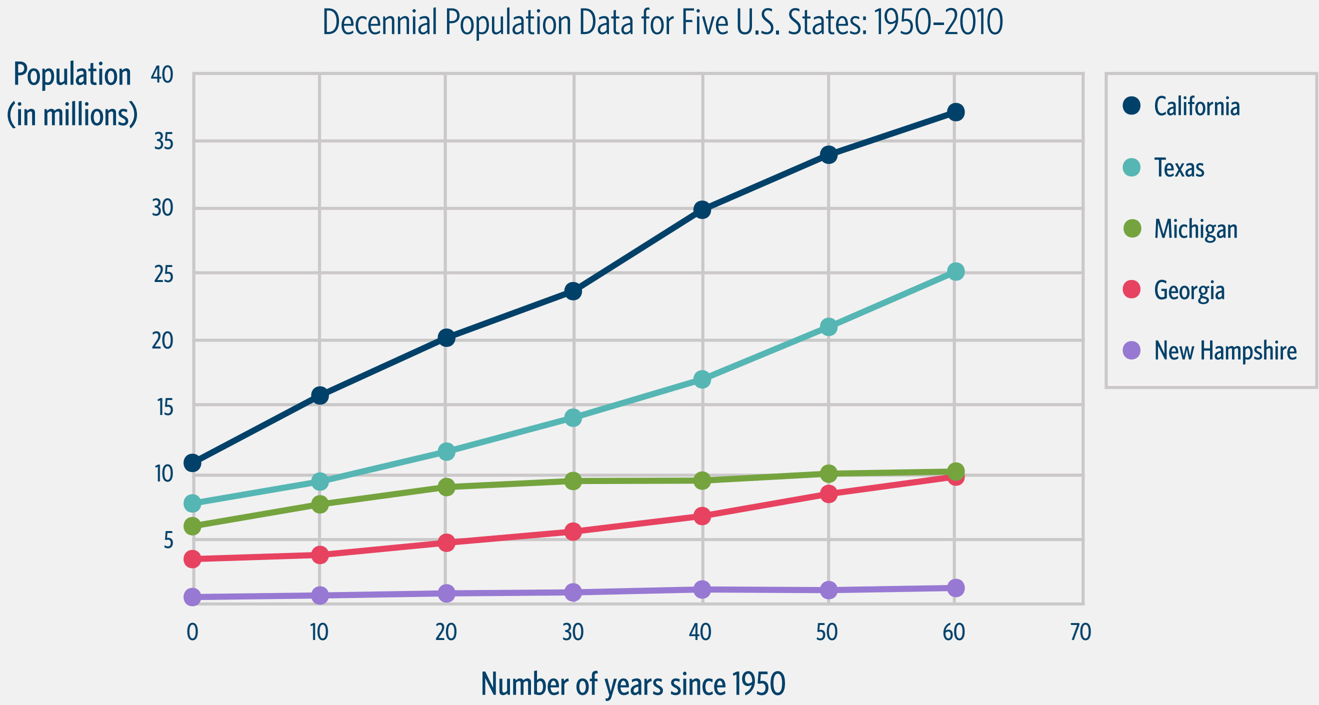
www.census.gov/2010census/data/apportionment-pop-text.php

Click on the link above to view the source data online.

In modeling, we sometimes change the format of numbers so they are easier to work with. Here are the same data written as rounded numbers in the hundred thousands:

	0	10	20	30	40	50	60
California	106	157	200	237	298	339	373
Georgia	34	39	46	55	65	82	97
Michigan	64	78	89	93	93	99	99
New Hampshire	5	6	7	9	11	12	13
Texas	77	96	112	142	170	209	251

Item 2: Five States' Growing Populations in a Graph: 1950-2010



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