

CALCULATING WITH SCIENTIFIC NOTATION – COMPARING POPULATIONS

TEACHER VERSION

Subject Level:
Middle School Math

Grade Level:
8

Approx. Time Required:
45–60 minutes

Learning Objectives:

- Students will be able to perform operations with numbers, including decimals, expressed in scientific notation.
- Students will be able to use scientific notation to express ratios.
- Students will be able to create and interpret a scatter plot.

Activity Description

Students will use population data from the U.S. Census Bureau to read and write numbers in scientific notation and to make comparisons of that data for states and decades. Students will also write ratios of the populations of their school's state in different decades, determine if the ratios show a pattern, and make a scatter plot of the data.

Suggested Grade Level:
8

Approximate Time Required:
45–60 minutes

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Topics:

- Scatter plots
- Scientific notation

Skills Taught:

- Creating and interpreting a scatter plot
- Investigating patterns of association
- Reading, writing, and calculating with scientific notation

Materials Required

- The student version of this activity, 8 pages
- Graphing calculators

Activity Item

The following item is part of this activity. The item, its data source, and instructions for viewing the source data online appear at the end of this teacher version.

- Item 1: Population of the Nation, All 50 States, the District of Columbia, and Puerto Rico: 1920–2020

For more information to help you introduce your students to the U.S. Census Bureau, read "*Census Bureau 101 for Students*." This information sheet can be printed and passed out to your students as well.

Standards Addressed

See charts below. For more information, read "*Overview of Education Standards and Guidelines Addressed in Statistics in Schools Activities*."

Common Core State Standards for Mathematics

Standard	Domain	Cluster
CCSS.MATH.CONTENT.8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.	8 EE – Expressions & Equations	Work with radicals and integer exponents.
CCSS.MATH.CONTENT.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	8 EE – Expressions & Equations	Work with radicals and integer exponents.

Standard	Domain	Cluster
CCSS.MATH.CONTENT.8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	8 SP – Statistics & Probability	Investigate patterns of association in bivariate data.

Common Core State Standards for Mathematical Practice

Standard
CCSS.MATH.PRACTICE.MP1. Make sense of problems and persevere in solving them. Students will determine which operation(s) to perform for different problems and follow through with the calculations to solve them.
CCSS.MATH.PRACTICE.MP4. Model with mathematics. Students will use numbers written in scientific notation to model various situations related to population and will create a scatter plot to model state population changes over time.
CCSS.MATH.PRACTICE.MP6. Attend to precision. Students will make accurate calculations, showing their work and explaining each step, and will check their answers using a calculator.

National Council of Teachers of Mathematics' Principles and Standards for School Mathematics

Content Standard	Students should be able to:	Expectation for Grade Band
Number and Operations	Understand numbers, ways of representing numbers, relationships among numbers, and number systems.	Develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation.

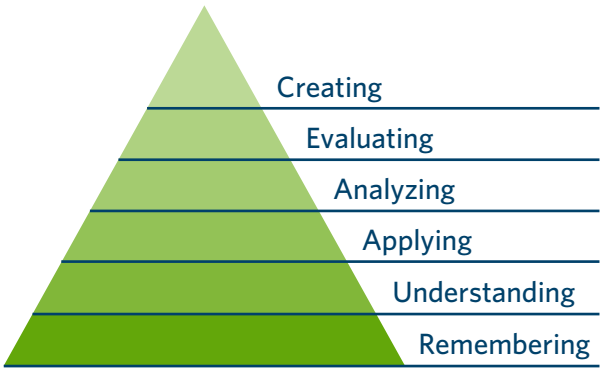
Content Standard	Students should be able to:	Expectation for Grade Band
Algebra	Use mathematical models to represent and understand quantitative relationships.	Model and solve contextualized problems using various representations, such as graphs, tables, and equations.
Data Analysis and Probability	Develop and evaluate inferences and predictions that are based on data.	Make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit.

Guidelines for Assessment and Instruction in Statistics Education

GAISE	Level A	Level B	Level C
Formulate Questions		X	
Collect Data			
Analyze Data		X	
Interpret Results		X	

Bloom's Taxonomy

Students will ***apply*** their knowledge of scientific notation to ***analyze*** population data.



Teacher Notes

Before the Activity

Students must understand the following key terms:

- **Exponential relationship** – the relationship of data points in a distribution that is best represented by a curved line, where one of the variables in the equation of that line is an exponent *($y = x^a$)*
- **Linear relationship** – the relationship of data points in a distribution that is best represented by a straight line *($y = mx + b$)*

Students should have the following skills:

- Ability to read, write, and perform calculations with numbers in scientific notation
- Ability to read and enter numbers in scientific notation on graphing calculators
- Ability to make a scatter plot using a calculator

Teachers should ask students to examine the data in **Item 1**, then prompt students' thinking by asking:

- How many people lived in our school's state in 2020? What about in 1920?
- How did the population of our school's state change between 1920 and 2020?
- How does the population of our school's state in 2020 compare with that of neighboring states or states in other parts of the country?

During the Activity

Teachers could divide students into groups of two to four to complete this activity.

Teachers should direct students to complete all scientific notation questions by hand and to create the scatter plot using their graphing calculators. Tutorials for using TI graphing technology are available here (*https://education.ti.com/en/professional-development/teachers-and-teams/online-learning/product-tutorials*).

Teachers should have students check their work with a calculator before creating their scatter plots in question 7.

Teachers could simplify this activity by formulating the questions for students in question 4. Teachers could also help visual learners by highlighting ahead of time the rows, columns, or specific numbers they need to use in **Item 1**.

After the Activity

Teachers should facilitate a discussion about students' findings from the questions they investigated in question 4, focusing on what students found most interesting. If students worked in groups, teachers could direct them to discuss their findings with their group and then with the rest of the class.

Extension Idea

- Teachers could have students use the population density information from the source data for **Item 1** (*www.census.gov/data/tables/time-series/dec/density-data-text.html*) to compare the population densities of different states in the same year or of one state in different years.

Student Activity

Click ***here*** to download a printable version for students.

Activity Item

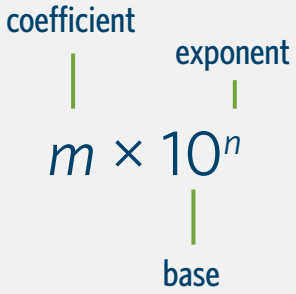
The following item is part of this activity and appears at the end of this student version.

- Item 1: Population of the Nation, All 50 States, the District of Columbia, and Puerto Rico: 1920–2020

Student Learning Objectives

- I will be able to perform operations with numbers, including decimals, expressed in scientific notation.
- I will be able to use scientific notation to express ratios.
- I will be able to create and interpret a scatter plot.

Refer to the following example of scientific notation and its parts to help you complete this activity. In scientific notation, the coefficient *(m)* is a number greater than or equal to 1 and less than 10, and the exponent *(n)* raises the base of 10 to a power.



1. Use **Item 1: Population of the Nation, All 50 States, the District of Columbia, and Puerto Rico: 1920–2020** to fill in the following table, rounding your scientific notations to two decimal places.

Location	Population	Scientific Notation
Total U.S. population in 1920	**106,021,568**	**1.06×10^8**
Population of Alabama in 1920	**2,348,174**	**2.35×10^6**
Population of Alaska in 1920	**55,036**	**5.50×10^4**
Population of Puerto Rico in 2020	**3,285,874**	**3.29×10^6**
Total U.S. population in 2020	**331,449,281**	**3.31×10^8**

2. Answer the following questions about what different scientific notations represent.
   a. 2.03×10^8 represents the population of the United States in what year? **1970**
   b. 2.92×10^6 represents the population of Tennessee in what year? **1940**
   c. 7.84×10^5 represents the population of Delaware in what year? **2000**

3. Answer the following questions and prompts to compare the populations of New Jersey and New Mexico in 2020. Round your scientific notations to two decimal places.
   a. Write the population of New Jersey in scientific notation.

      $9{,}288{,}994 = 9.29 \times 10^6$

   b. Write the population of New Mexico in scientific notation.

      $2{,}117{,}522 = 2.12 \times 10^6$

   c. Subtract the 2020 New Mexico population from the 2020 New Jersey population written in scientific notation, showing your work. (To subtract using scientific notation, first check that your two bases use the same exponent. If they do, subtract the coefficients and leave the exponent the same. If they don't, convert one of the coefficients so that both use the same exponent, then subtract the coefficients and leave the exponent the same.)

      $(9.29 - 2.12) \times 10^6 = 7.17 \times 10^6$

   d. Use your answer for question 3c to indicate how many more people lived in New Jersey than in New Mexico in 2020:

      New Jersey had **7.17×10^6, or about 7 million** more residents than New Mexico in 2020.

e. Divide the New Jersey population by the New Mexico population, using the values written in scientific notation and showing your work. (To divide with scientific notation, divide the coefficients and subtract their exponents.)

$(9.29 / 2.12) \times 10^{6-6} = 4.38 \times 10^{0}$

f. Indicate how many times more people lived in New Jersey than in New Mexico in 2020:

New Jersey had **4.38×10^{0}, or about 4** times more residents than New Mexico in 2020.

4. Now write three of your own questions that each compare two population values: either for two different states in the same year or for the same state in two different years.

Question 1

a. Write your question.
b. Write your two populations in scientific notation, rounding to two decimal places.
c. Complete your calculation, showing all your work.
d. Write a concluding statement.

Question 2

a. Write your question.
b. Write your two populations in scientific notation, rounding to two decimal places.
c. Complete your calculation, showing all your work.
d. Write a concluding statement.

Question 3

a. Write your question.
b. Write your two populations in scientific notation, rounding to two decimal places.
c. Complete your calculation, showing all your work.
d. Write a concluding statement.

Student responses will vary; an example is:

a. **Between Alabama and Arizona, which state had the larger population in 1950, and how much larger was it?**
b. **Alabama: 3.06×10^{6}; Arizona: 7.50×10^{5}**
c. **$3.06 \times 10^{6} - 7.50 \times 10^{5} = 30.6 \times 10^{5} - 7.50 \times 10^{5} = 23.1 \times 10^{5} = 2.31 \times 10^{6}$**
d. **Alabama had about 2,310,000 (or 2.31×10^{6}) more residents than did Arizona in 1950.**

5. Use data for your school's state to calculate the ratios of its population in one decade to its population in the previous decade, using scientific notation. Round your answers to two decimal places and show your work.

   Student answers will vary depending on the school's state; sample answers below use Arizona.

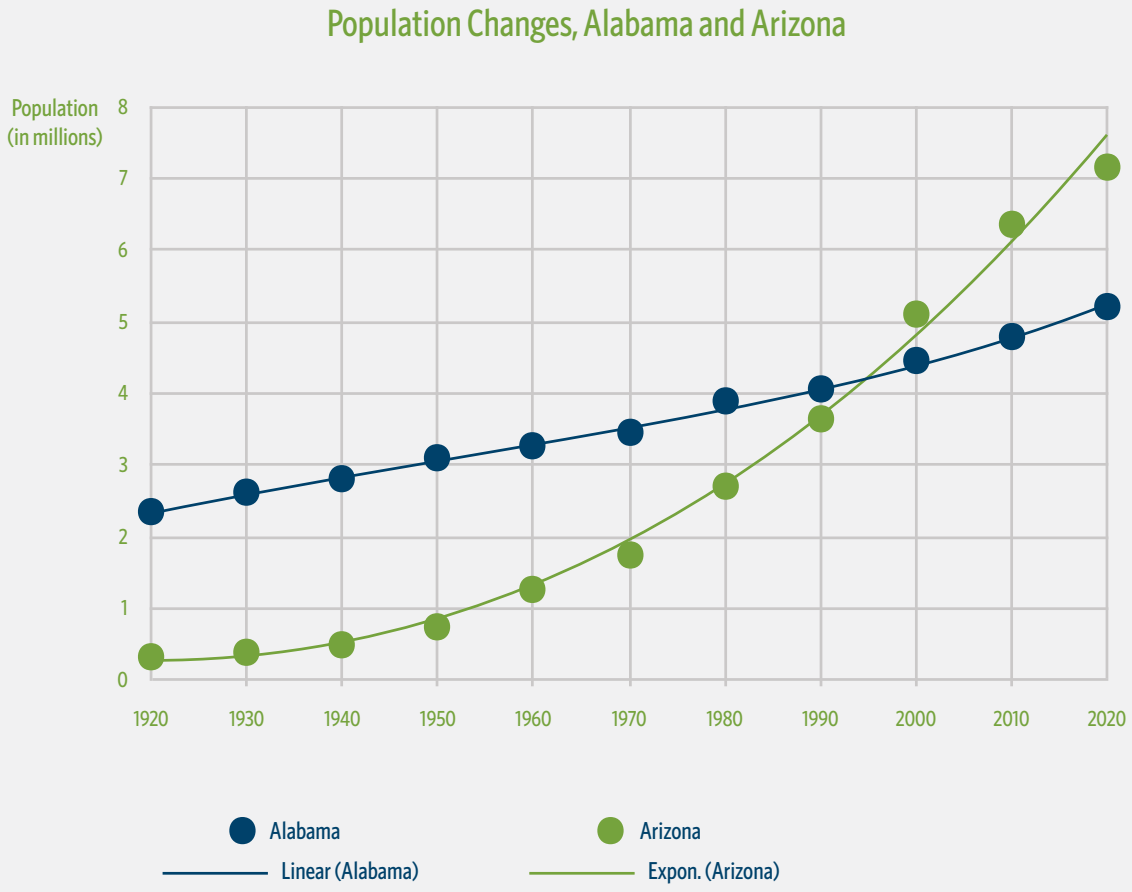
Ratio Formulas by Population Year	Ratio in Scientific Notation
2020 population / 2010 population =	Example: $7.15 \times 10^6 / 6.39 \times 10^6 = 1.12$
2010 population / 2000 population =	**$6.39 \times 10^6 / 5.13 \times 10^6 = 1.25$**
2000 population / 1990 population =	**$5.13 \times 10^6 / 3.67 \times 10^6 = 1.40$**
1990 population / 1980 population =	**$3.67 \times 10^6 / 2.72 \times 10^6 = 1.35$**
1980 population / 1970 population =	**$2.72 \times 10^6 / 1.77 \times 10^6 = 1.54$**
1970 population / 1960 population =	**$1.77 \times 10^6 / 1.30 \times 10^6 = 1.36$**
1960 population / 1950 population =	**$1.30 \times 10^6 / 7.50 \times 10^5 = 1.73$**
1950 population / 1940 population =	**$7.50 \times 10^5 / 4.99 \times 10^5 = 1.50$**
1940 population / 1930 population =	**$4.99 \times 10^5 / 4.36 \times 10^5 = 1.14$**
1930 population / 1920 population =	**$4.36 \times 10^5 / 3.34 \times 10^5 = 1.31$**

6. Do you see a pattern in the ratios? If so, what is it?

   Student answers will vary depending on the school's state. For Arizona, the population increases with each decade. The rounded ratios are all between 1.14 and 1.73.

7. Using a graphing calculator, make a scatter plot that shows the population of your school's state in each decade, from 1920 to 2020.

   Student graphs will vary depending on the school's state. Below is a sample scatter plot for teachers showing the populations over time for both Alabama and Arizona, including a trendline and other details for teachers' convenience.



a. Does the graph show a positive or negative association between the population of your school's state and the decade? Is there a linear, exponential, or other type of relationship? Explain.

Student answers will vary depending on the school's state. For Arizona, the graph shows a positive association and an approximately exponential relationship.

b. Looking at your graph, predict the population of your school's state last year.

Student answers will vary depending on the state selected and the year.

If students have access to Microsoft Excel, teachers could let them know that they can re-create their graph there and "format the trendline" to help them make their prediction. Teachers could also direct students to check the accuracy of their predictions by going here (*www.census.gov/quickfacts/*) and finding the Census Bureau estimate of the school's state population for last year.

Item 1: Population of the Nation, All 50 States, the District of Columbia, and Puerto Rico: 1920–2020

State/District	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010	2020
Alabama	2,348,174	2,646,248	2,832,961	3,061,743	3,266,740	3,444,165	3,893,888	4,040,587	4,447,100	4,779,736	5,024,279
Alaska	55,036	59,278	72,524	128,643	226,167	300,382	401,851	550,043	626,932	710,231	733,391
Arizona	334,162	435,573	499,261	749,587	1,302,161	1,770,900	2,718,215	3,665,228	5,130,632	6,392,017	7,151,502
Arkansas	1,752,204	1,854,482	1,949,387	1,909,511	1,786,272	1,923,295	2,286,435	2,350,725	2,673,400	2,915,918	3,011,524
California	3,426,861	5,677,251	6,907,387	10,586,223	15,717,204	19,953,134	23,667,902	29,760,021	33,871,648	37,253,956	39,538,223
Colorado	939,629	1,035,791	1,123,296	1,325,089	1,753,947	2,207,259	2,889,964	3,294,394	4,301,261	5,029,196	5,773,714
Connecticut	1,380,631	1,606,903	1,709,242	2,007,280	2,535,234	3,031,709	3,107,576	3,287,116	3,405,565	3,574,097	3,605,944
Delaware	223,003	238,380	266,505	318,085	446,292	548,104	594,338	666,168	783,600	897,934	989,948
District of Columbia	437,571	486,869	663,091	802,178	763,956	756,510	638,333	606,900	572,059	601,723	689,545
Florida	968,470	1,468,211	1,897,414	2,771,305	4,951,560	6,789,443	9,746,324	12,937,926	15,982,378	18,801,310	21,538,187
Georgia	2,895,832	2,908,506	3,123,723	3,444,578	3,943,116	4,589,575	5,463,105	6,478,216	8,186,453	9,687,653	10,711,908
Hawaii	255,912	368,336	423,330	499,794	632,772	768,561	964,691	1,108,229	1,211,537	1,360,301	1,455,271
Idaho	431,866	445,032	524,873	588,637	667,191	712,567	943,935	1,006,749	1,293,953	1,567,582	1,839,106
Illinois	6,485,280	7,630,654	7,897,241	8,712,176	10,081,158	11,113,976	11,426,518	11,430,602	12,419,293	12,830,632	12,812,508
Indiana	2,930,390	3,238,503	3,427,796	3,934,224	4,662,498	5,193,669	5,490,224	5,544,159	6,080,485	6,483,802	6,785,528
Iowa	2,404,021	2,470,939	2,538,268	2,621,073	2,757,537	2,824,376	2,913,808	2,776,755	2,926,324	3,046,355	3,190,369
Kansas	1,769,257	1,880,999	1,801,028	1,905,299	2,178,611	2,246,578	2,363,679	2,477,574	2,688,418	2,853,118	2,937,880
Kentucky	2,416,630	2,614,589	2,845,627	2,944,806	3,038,156	3,218,706	3,660,777	3,685,296	4,041,769	4,339,367	4,505,836
Louisiana	1,798,509	2,101,593	2,363,880	2,683,516	3,257,022	3,641,306	4,205,900	4,219,973	4,468,976	4,533,372	4,657,757
Maine	768,014	797,423	847,226	913,774	969,265	992,048	1,124,660	1,227,928	1,274,923	1,328,361	1,362,359
Maryland	1,449,661	1,631,526	1,821,244	2,343,001	3,100,689	3,922,399	4,216,975	4,781,468	5,296,486	5,773,552	6,177,224
Massachusetts	3,852,356	4,249,614	4,316,721	4,690,514	5,148,578	5,689,170	5,737,037	6,016,425	6,349,097	6,547,629	7,029,917
Michigan	3,668,412	4,842,325	5,256,106	6,371,766	7,823,194	8,875,083	9,262,078	9,295,297	9,938,444	9,883,640	10,077,331
Minnesota	2,387,125	2,563,953	2,792,300	2,982,483	3,413,864	3,804,971	4,075,970	4,375,099	4,919,479	5,303,925	5,706,494
Mississippi	1,790,618	2,009,821	2,183,796	2,178,914	2,178,141	2,216,912	2,520,638	2,573,216	2,844,658	2,967,297	2,961,279
Missouri	3,404,055	3,629,367	3,784,664	3,954,653	4,319,813	4,676,501	4,916,686	5,117,073	5,595,211	5,988,927	6,154,913
Montana	548,889	537,606	559,456	591,024	674,767	694,409	786,690	799,065	902,195	989,415	1,084,225
Nebraska	1,296,372	1,377,963	1,315,834	1,325,510	1,411,330	1,483,493	1,569,825	1,578,385	1,711,263	1,826,341	1,961,504
Nevada	77,407	91,058	110,247	160,083	285,278	488,738	800,493	1,201,833	1,998,257	2,700,551	3,104,614
New Hampshire	443,083	465,293	491,524	533,242	606,921	737,681	920,610	1,109,252	1,235,786	1,316,470	1,377,529
New Jersey	3,155,900	4,041,334	4,160,165	4,835,329	6,066,782	7,168,164	7,364,823	7,730,188	8,414,350	8,791,894	9,288,994
New Mexico	360,350	423,317	531,818	681,187	951,023	1,016,000	1,302,894	1,515,069	1,819,046	2,059,179	2,117,522
New York	10,385,227	12,588,066	13,479,142	14,830,192	16,782,304	18,236,967	17,558,072	17,990,455	18,976,457	19,378,102	20,201,249

Item 1: Population of the Nation, All 50 States, the District of Columbia, and Puerto Rico: 1920–2020 (Continued)

State/District	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010	2020
North Carolina	2,559,123	3,170,276	3,571,623	4,061,929	4,556,155	5,082,059	5,881,766	6,628,637	8,049,313	9,535,483	10,439,388
North Dakota	646,872	680,845	641,935	619,636	632,446	617,761	652,717	638,800	642,200	672,591	779,094
Ohio	5,759,394	6,646,697	6,907,612	7,946,627	9,706,397	10,652,017	10,797,630	10,847,115	11,353,140	11,536,504	11,799,448
Oklahoma	2,028,283	2,396,040	2,336,434	2,233,351	2,328,284	2,559,229	3,025,290	3,145,585	3,450,654	3,751,351	3,959,353
Oregon	783,389	953,786	1,089,684	1,521,341	1,768,687	2,091,385	2,633,105	2,842,321	3,421,399	3,831,074	4,237,256
Pennsylvania	8,720,017	9,631,350	9,900,180	10,498,012	11,319,366	11,793,909	11,863,895	11,881,643	12,281,054	12,702,379	13,002,700
Rhode Island	604,397	687,497	713,346	791,896	859,488	946,725	947,154	1,003,464	1,048,319	1,052,567	1,097,379
South Carolina	1,683,724	1,738,765	1,899,804	2,117,027	2,382,594	2,590,516	3,121,820	3,486,703	4,012,012	4,625,364	5,118,425
South Dakota	636,547	692,849	642,961	652,740	680,514	665,507	690,768	696,004	754,844	814,180	886,667
Tennessee	2,337,885	2,616,556	2,915,841	3,291,718	3,567,089	3,923,687	4,591,120	4,877,185	5,689,283	6,346,105	6,910,840
Texas	4,663,228	5,824,715	6,414,824	7,711,194	9,579,677	11,196,730	14,229,191	16,986,510	20,851,820	25,145,561	29,145,505
Utah	449,396	507,847	550,310	688,862	890,627	1,059,273	1,461,037	1,722,850	2,233,169	2,763,885	3,271,616
Vermont	352,428	359,611	359,231	377,747	389,881	444,330	511,456	562,758	608,827	625,741	643,077
Virginia	2,309,187	2,421,851	2,677,773	3,318,680	3,966,949	4,648,494	5,346,818	6,187,358	7,078,515	8,001,024	8,631,393
Washington	1,356,621	1,563,396	1,736,191	2,378,963	2,853,214	3,409,169	4,132,156	4,866,692	5,894,121	6,724,540	7,705,281
West Virginia	1,463,701	1,729,205	1,901,974	2,005,552	1,860,421	1,744,237	1,949,644	1,793,477	1,808,344	1,852,994	1,793,716
Wisconsin	2,632,067	2,939,006	3,137,587	3,434,575	3,951,777	4,417,731	4,705,767	4,891,769	5,363,675	5,686,986	5,893,718
Wyoming	194,402	225,565	250,742	290,529	330,066	332,416	469,557	453,588	493,782	563,626	576,851
Puerto Rico	1,299,809	1,543,913	1,869,255	2,210,703	2,349,544	2,712,033	3,196,520	3,522,037	3,808,610	3,725,789	3,285,874
United States	106,021,568	123,202,660	132,165,129	151,325,798	179,323,175	203,211,926	226,545,805	248,709,873	281,421,906	308,745,538	331,449,281

www.census.gov/data/tables/time-series/dec/density-data-text.html

Click on the link above to view the source data online.